The Rate of Reserve Requirements and Monetary Policy in Uruguay: a DSGE Approach

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Abstract

The goal of this paper is analyze the role of reserve requirements in the transmission of the monetary policy in Uruguay. Motivated by recent changes in the banking reserves rules implemented by the Central Bank a DSGE model that includes this feature is developed to understand its effects in the macroeconomic variables. The model is an extended version of the new keynesian business cycle model developed in Cubas (2011). It includes a banking system and a richly modeled monetary policy with inflation targeting with interventions in the foreign exchange market by the central bank. The model is then calibrated to Uruguay and then used to analyze the reaction of the macroeconomic variables to exogenous shocks to productivity, government expenditures, international interest rate, monetary interventions, risk premium, international inflation and price of exports; under different reserve requirement rules. The model predicts minor changes in the response of the variables of interest for the observed changes in the mean rate of reserve requirements in Uruguay in the period 2005-2011.

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1 Introduction

Motivated by recent changes in the banking reserves rules implemented by the Central Bank a DSGE model is developed in order to understand its effects in macroeconomic variables. Its main feature is that it includes a banking system and a richly modeled monetary policy with inflation targeting with interventions in the foreign exchange market by the central bank and an exogenous rate of reserve requirements set by the monetary authority. The main goal of this paper is to use the model to analyze the role of reserve requirements in the transmission of the monetary policy in Uruguay.

The model is an extended version of the new keynesian business cycle model developed in Cubas (2011) calibrated to Uruguay in 2005. The economy is a small open economy populated with households, a domestic production sector that produce final consumption goods combining domestic intermediate inputs produced by the intermediate domestic sector. In addition, there are firms that operate with the rest of the world. First, there are firms that produce primary goods for export. Second, firms that import final consumption goods and, finally the firms that import the intermediate inputs mentioned above.

There is a banking system that hold deposits from households and get funding from the rest of the world. An exogenous portion of the amount of total deposits is hold by the central bank as reserves which are non-remunerated. They lend to domestic firms and buy central bank short term bonds. The public sector is composed by a central bank and a central government.

We model a central bank that closely mimics the type of policy carried out by the Central Bank of Uruguay: inflation targeting policy and interventions in the foreign exchange market. Specifically, the central bank issues both the currency demanded by the households and domestic short term bonds demanded by domestic banks, and holds international reserves in the form of riskless bond issued abroad. The central government is simple modeled as it just just finance an exogenous stream of expenditures with lump sum taxes. Monetary policy is modeled as a inflation targeting regime with managed exchange rate floating. Therefore, the central bank intervenes both in the money market and in the foreign exchange market to aim for a operational target for the interest rate and to smooth out exchange rate movements. In line with the literature, price stickiness is also introduced in the form of Calvo pricing with full indexation.

When the model is linearized and solved it reduces to only six dynamic equations: a IS curve, a Phillips equation, balance of payments, risk-adjusted interest rate parity equation and the two policy equations: the interventions of the central bank in the money and foreign exchange markets. We further carry an empirical analysis by calibrating the model to the Uruguayan economy in 2005, we call it the benchmark economy.

Finally, we show the impulse response analysis when the model economy is hit by exogenous
shocks to: the demand for consumption goods, productivity, price of export goods, international inflation, international interest rate, government expenditure and interest rate risk premium both in the benchmark case and in two hypothetical economies: one with the maximum rate of reserve requirements observed in Uruguay in the period 2005-2012 (22%) and one with a reserve requirements rate of 50%. Interestingly, we only observe minor changes in the macroeconomic variables for the rates observed in Uruguay in the period analyzed.

In the last few years we have experienced the development of a new set of dynamic general equilibrium models with micro foundations aimed to help policymakers to better understand the behavior of endogenous macroeconomic relations we observe in the data. These models are based on the standard real business cycle models of the eighties but, in order to better mimic the data and to be useful for policy analysis they have been modified to add various frictions and shocks that make them more realistic and at the same time more complex. Important papers in the literature are Christiano, Echeibaum and Evans (2005), Smets and Wouters (2003) for the case of developed countries and, in the case of of Latin American countries we have Escude (2007) and Castillo, Montoro and Tuesta (2009). This work aims to use these modern tools to model the Uruguayan economy and contribute to the understanding of the effects of monetary policy.

2 The Model

The economy is populated by three types of agents: households, firms and government.

2.1 Households

There is a continuum of infinitely lived households indexed by \( h \in [0, 1] \), which represents the type of differentiated labor they supply to the market in which they compete monopolistically.

In addition to the state contingent assets household hold domestic currency, \( M^0_t \), so that they economize transaction costs. It is assumed that transaction cost are a decreasing and convex function of the currency-consumption ratio:

\[
\tau_M(\omega_t), \quad \tau'_M < 0 \quad \text{and} \quad \tau''_M > 0
\]

(1)

with

\[
\omega_t = \frac{M^0_t(h)}{P_t C_t(h)}
\]

(2)
where $C_t$ is the consumption bundle (composed by domestic and imported goods), $P_t$ is its price index and $P_t^C$ the price index of domestic consumption. We can rewrite (2) as

$$\omega_t = \frac{m_t^0(h)}{p_t^C C_t(h)}$$

(3)

where $m_t^0 = M_t^0/P_t$ are cash holdings in real terms and, $p_t = P_t^C/P_t$ the relative price of domestic consumption.

Households also hold one period local currency deposits, $D_t$ in domestic banks which pay interest $i_t$. This is a risk-free interest rate since we assume deposits are perfectly insured by the Central Bank. In addition, it is assumed that they cannot incur in debt and so $D_t > 0$.

Households own the firms and they also supply their exclusive labor type $h_t$ (with an upper-bound $\overline{h}$) in monopolistic competition and so in setting their wages they take the aggregate wage ($W_t$) and labor supply as given. They also have their labor-type demand as a constraint in the wage determination:

$$h_t(h) = h_t \left( \frac{W_t(h)}{W_t} \right)^{-\psi}$$

(4)

with the aggregate wage index given by

$$W_t = \left\{ \int_0^{\infty} W_t(h)^{1-\psi} dh \right\}^{1/(1-\psi)}$$

(5)

where $\psi$ is the elasticity of substitution between labor types.

Household $h$ derive utility from the consumption of domestic and imported goods and, leisure and maximizes expected lifetime utility given by,

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ z_{t+j}^C C_t^{1-\sigma} (h) + \left[ h_{t+j}^{1+\chi} (h) \right] + \left[ \frac{h_{t+j}^{1+\chi} (h)}{1 + \chi} \right] \right\}$$

where $\mathbb{E}_t$ is the expectation operator in $t$, $\beta$ is the discount factor and $z_{t+j}^C$ represents a demand shock which is common to all households.

The aggregate consumption bundle is a CES aggregate of domestic and imported goods:

$$C_t = \left( a_D^\frac{1}{\theta_C} (C_t^D)^{\frac{\theta_C-1}{\theta_C}} + a_N^\frac{1}{\theta_C} (C_t^N)^{\frac{\theta_C-1}{\theta_C}} \right)^{\frac{\theta_C}{\theta_C-1}}$$

(7)

with $a_D + a_N = 1$ and $\theta_C$ being the elasticity of substitution between the two types of consumption goods. Additionally, both imported and domestic consumption goods are CES aggregates of $i$ varieties of goods:
\[ C^D_t = \left( \int_0^1 C^D_t(i) \frac{\partial C^D_t}{\partial i} \, di \right)^{\frac{\theta}{\theta - 1}}, \quad \theta > 1 \]  

(8)

\[ C^N_t = \left( \int_0^1 C^N_t(i) \frac{\partial C^N_t}{\partial i} \, di \right)^{\frac{\theta_N}{\theta_N - 1}}, \quad \theta_N > 1. \]  

(9)

Therefore, total consumption expenditure is given by

\[ P^C_t C_t = P^D_t C^D_t + P^N_t C^N_t. \]  

(10)

Therefore, for a given level of aggregate consumption \( C_t \), by minimizing (10) subject to (7) we get the price indexes for both types of consumption goods and for the aggregate consumption bundle:

\[ P^D_t = a_D^1 \frac{P^C_t}{C_t} \left( \frac{C^D_t}{C^D_t} \right)^{-\frac{1}{\theta_C}}, \]  

(11)

\[ P^N_t = a_N^1 \frac{P^C_t}{C_t} \left( \frac{C^N_t}{C^N_t} \right)^{-\frac{1}{\theta_C}}, \]  

(12)

\[ P^C_t = \left( a_D (P_t)^{1-\theta_C} + a_N (P^N_t)^{1-\theta_C} \right)^{1/\theta_C}. \]  

(13)

where

\[ a_D = \frac{P^D_t C_t}{P^C_t C_t}, \quad a_N = 1 - a_D = \frac{P^N_t C^N_t}{P^C_t C_t}. \]  

(14)

Similarly, we obtain the price levels for each variety of the goods that compose each type of good:

\[ P_t(i) = P_t \left( \frac{C^D_t(i)}{C^D_t} \right)^{-\frac{1}{\theta_C}}, \]  

(15)

\[ P^N_t(i) = P^N_t \left( \frac{C^N_t(i)}{C^N_t} \right)^{-\frac{1}{\theta_C}}. \]  

(16)

Finally, from (13) and (14) we can obtain an expression for the consumption inflation rate:
\[
\eta_t^C = \left[ \frac{a_D}{a_D + a_N(p_{t-1}^N)^{1-\theta_C}}(\pi_t)^{1-\theta_C} + \left( 1 - \frac{a_D}{a_D + a_N(p_{t-1}^N)^{1-\theta_C}} \right) (\pi_t^N)^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}. \tag{17}
\]

**Household Maximization Problem**  Household \( h \) chooses streams of consumption \( C_t(h) \), deposits \( D_t(h) \), cash holdings \( M_t(h) \) and \( W_t(h) \) so that it maximizes 6 subject to its labor demand constraint given by (4) and its period by period budget constraint which reads

\[
\frac{M_0^t(h)}{p_t} + \frac{D_t(h)}{p_t} = \frac{\Pi_t(h)}{p_t} h_t(h) - \frac{T_t(h)}{p_t} + Y_t + \left[ \frac{\lambda_t}{\lambda_t + 1} \right] p_t^C C_t(h)
\]

where \( \Pi_t \) are profits household receive from the firms they own in nominal terms, \( T_t(h) \) are lump sum taxes net of transfers and, \( Y_t(h) \) is the income coming from state contingent asset holdings.

The first order conditions are the following:

\[
C_t: \quad \varepsilon_t^C C_t^{-\sigma} = \lambda_t \phi_M \left( \frac{m_0^t}{p_t^C C_t} \right) \tag{18}
\]

\[
D_t: \quad \lambda_t = \beta (1+i_t) E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right) \tag{19}
\]

\[
M_{t-1}^0, H: \quad \lambda_t \left[ 1 + \tau_M \left( \frac{m_0^t}{p_t^C C_t} \right) \right] = \beta E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right) \tag{20}
\]

\[
W_t: \quad \frac{W_t}{p_t} = \frac{\psi h_t^\chi}{\psi - 1} \lambda_t \tag{21}
\]

plus the transversality condition \( \lim_{t \to \infty} \beta^t D_t = 0 \).

The variable \( \lambda_t \) is the lagrange multiplier, \( \pi_t \) is the inflation rate of the domestic good bundle, i.e. \( \pi_t = P_{t+1}/P_t \).

The function \( \phi(.) \) gives the total effect on expenditures of a marginal increase in consumption and we assume takes the following form:

\[
\phi_M(\omega_t) = 1 + \tau_M(\omega_t) - \omega_t \tau_M'(\omega_t), \tag{22}
\]

so
\[ \varphi'_M(\omega_t) = \tau'_M(\omega_t) - \omega_t \tau''_M(\omega_t) < 0. \quad (23) \]

Therefore, (18) shows that the marginal utility gain from consuming one additional unit of consumption equals the foregone marginal utility of real income. Equation (19) states that the marginal utility cost of holding a unit of deposits must equal the discounted expected marginal utility from the returns it generates. In addition, equation (20) shows that the marginal cost of holding currency taking into account the saving in transaction costs it generates equals the expected marginal utility of using it next period where the purchase power is corrected by expected inflation. Moreover, by combining (19) and (20) we get

\[- \tau'_M \left( \frac{m^0_t}{p_t^C C_t} \right) = 1 - \frac{1}{1 + i_t} \quad (24)\]

which shows that the optimal stock of currency as a fraction of expenditure is such that the reduction of transaction costs generated by money holdings must equal the opportunity cost of holding money. Providing the function \( \tau' \) is invertible, by inverting it we obtain the demand for cash

\[ m^0_t = L(1 + i_t) p_t^C C_t \quad (25) \]

where the function \( L \) is

\[ L(1 + i_t) \equiv \left( \tau'_M(,) \right)^{-1} \left( 1 - \frac{1}{1 + i_t} \right), \quad (26) \]

and so its derivative reads

\[ L'(1 + i_t) = \left[ \tau''_M(,) (1 + i_t)^2 \right]^{-1} < 0. \quad (27) \]

To facilitate notation it proves convenient to use the following auxiliary functions

\[ \bar{\varphi}(1 + i_t) \equiv \varphi(L(1 + i_t)), \quad (28) \]

and

\[ \bar{\tau}_M(1 + i_t) \equiv \tau_M(L(1 + i_t)). \quad (29) \]

In addition, from (18), (19) and (21) we obtain

\[ \beta(1 + i_t) E_t \left( \frac{z^{C}_{t+1}}{z^C_t} \right) = E_t \left( \left( \frac{C_{t+1}}{C_t} \right) \sigma \frac{\bar{\varphi}_M(1 + i_{t+1})}{\bar{\varphi}_M(1 + i_t)} \pi_{t+1} \right) \quad (30) \]
and
\[ \frac{W_t}{P_t} = \frac{\psi}{\psi - 1} h_t^X C_t^\sigma \varphi_M (1 + i_{t+1}) / \zeta_t^C. \] (31)

Equation (30) is the standard Euler equation in which the marginal utility of extra units of consumption is adjusted by the transaction costs and the inflation rate. Finally, equation (31) determines the labor supply and the real wage which is a markup (recall each labor type supplies its labor in a monopolistically competitive market) over the marginal rate of substitution between labor and consumption. Note that the more substitutes are labor types the lesser the markup.

2.2 Firms

2.2.1 Domestic Final Goods Producers

There are firms operating in competitive markets that produce a domestic final good \( Q_t \) by combining a continuum of intermediate goods indexed by \( i \) \( \tilde{Q}_t(i) \) produced by intermediate goods producers, according to the following CES technology:

\[ Q_t = \left( \int_0^1 \tilde{Q}_t(i) \frac{a-1}{\sigma} di \right)^{\frac{\theta}{(\theta - 1)}}, \] (32)

where \( \tilde{Q}_t(i) \) is the output of the intermediate firm producing the intermediate good \( i \) and \( \theta \) is the elasticity of substitution between intermediate inputs, with \( \theta > 1 \).

The representative firm solves the following profit maximization problem:

\[ \max_{\tilde{Q}_t(i)} P_t \left( \int_0^1 \tilde{Q}_t(i) \frac{a-1}{\sigma} di \right)^{\frac{\theta}{(\theta - 1)}} - \int_0^1 P_t(i) \tilde{Q}_t(i) di \] (33)

which renders the demand for each variety of intermediate input,

\[ \tilde{Q}_t(i) = Q_t \left( \frac{P_t(i)}{P_t} \right)^{-\theta}. \] (34)

By combining (32) and (34) we obtain the price index of domestic goods:

\[ P_t = \left( \int_0^1 P_t(i)^{1-\theta} di \right)^{-\frac{1}{1-\theta}}. \] (35)

2.2.2 Domestic Intermediate Goods Producers

There is a continuum of monopolistically competitive firms that produce intermediate goods by using labor and imported intermediate goods \( N^D \). We further assume there is no entry and exit
of firms in this sector of the economy. The production function of firm \( i \) which produces variety \( i \) reads:

\[
Q_t(i) = \varepsilon_t(h_t(i))^{bQ} (N_t^D(i))^{1-bQ}.
\]

(36)

where \( \varepsilon_t \) is a productivity shock that is common to all firms in the sector, \( bQ \) is the share of labor which is a CES aggregation of labor types \( h \):

\[
h_t(i) = \left( \int_0^1 h_t(h,i) \frac{w}{\psi} dh \right)^{\frac{\psi}{(1-\psi)}}.
\]

(37)

We assume that firms need working capital at the end of period \( t-1 \) in order to carry their production plans at period \( t \). In this way we assume a portion \( \varsigma \) of the firm’s cost bill is financed through loan obtained in the domestic bank system at rate \( i_t^L \). In this way changes in interest affect the demand of inputs and so aggregate output. This is modeled as in Neumeyer and Perri (2005), although these authors only considered that firms need loans to finance the wage bill only. Therefore, the variable costs for firm \( i \) is

\[
(1 + \varsigma_t^{l_{t-1}})[W_t h_t(i) + P_t^N N_t^D(i)].
\]

(38)

In this context the firm maximizes profits by minimizing costs which can be further analyzed separately. We first consider the minimization of total labor costs which means minimizing

\[
\int_0^1 W_t(h) h_t(h,i) dh
\]

subject to (37).

By solving the problem we obtain the inverse demand function for labor type \( h \) of firm \( i \):

\[
W_t(h) = W_t \left( \frac{h_t(h,i)}{h_t(i)} \right)^{-\frac{1}{\psi}}.
\]

(40)

Note that this labor demand equation is the same we take as given in equation (4) when stating the household maximization problem.

By adding the demand of all firms for labor type \( h \) we get

\[
h_t(h) = \int_0^1 h_t(h,i) di,
\]

(41)

and by further adding across households we get the labor input bundle

\[
h_t = \int_0^1 h_t(h) dh.
\]

(42)
We now turn to minimize total variable costs by solving the following minimization problem:

$$\min_{h_t(i),N_{D}^i(i)} \left\{ (1 + \zeta_i h_{t-1}^L) [W_t h_t(i) + P_t^N N_t^D(i)] \right\}$$  \hspace{1cm} (43)$$

subject to (34), where $Q_t(i)$ is taken as given.

By solving the problem we obtain the following first order conditions:

$$(1 + \zeta_i h_{t-1}^L) W_t h_t = b^q MC_t Q_t$$  \hspace{1cm} (44)$$

and

$$(1 + \zeta_i N_{D}^i(i)) P_t^N N_{D}^t = (1 - b^q) MC_t Q_t$$  \hspace{1cm} (45)$$

where $MC_t$ is the Lagrange multiplier. By adding these two equations and dividing them by the price level $P_t$ we get

$$(1 + \zeta_i h_{t-1}^L) (w_t h_t + p_t^N N_{D}^t) = mc_t Q_t$$  \hspace{1cm} (46)$$

where $w_t$ is the real wage, $mc_t$ is the real marginal cost and, $p_t^N$ represents the internal terms of trade of this small open economy.

By using (44) and (45) together with (36) we get the following expression for the real marginal cost:

$$mc_t = \frac{1}{\varepsilon_t \kappa} (1 + \zeta_i h_{t-1}^L) (w_t)^{(1 - b^q)} (p_t^N)^{(1 - b^q)}$$  \hspace{1cm} (47)$$

where $\kappa = (b^q)^{b^q} ((1 - b^q))^{(1 - b^q)}$.

We now obtain aggregate demand functions for $h_t$ and $N_{D}^t$ by using (44), (45) and (47):

$$h_t = \frac{1}{\varepsilon_t \kappa} b^q \left( \frac{p_t^N}{w_t} \right)^{(1 - b^q)} Q_t$$  \hspace{1cm} (48)$$

and

$$N_{D}^t = \frac{1}{\varepsilon_t \kappa} (1 - b^q) \left( \frac{w_t}{p_t^N} \right)^{b^q} Q_t.$$  \hspace{1cm} (49)$$

In order to obtain the demand for bank loans, we first divide (44) by (45) to get:

$$w_t h_t = \frac{b^q}{1 - b^q} p_t^N N_{D}^t.$$  \hspace{1cm} (50)$$

Using the first order conditions and (50) we get the expression for the bank’s loans demand

$$\frac{L_t}{P_t} = \zeta E_t (w_{t+1} h_{t+1} + p_t^N N_{D}^t) = \frac{\zeta}{b^q} E_t (w_{t+1} h_{t+1}).$$  \hspace{1cm} (51)$$
2.2.3 Firms Price Setting

The firm’s price setting is modeled as in Calvo (1983) with full indexation (Christiano, Eichenbaum and Evans (2001)). When making the price setting decision firms take the aggregate price and quantity level as given. Each period the firm has a probability \((1 - \alpha)\) of being able to set the optimal price for the good it sells. When it is not possible to set the optimal price, which is an event that occurs with probability \(\alpha\), they adjust prices by fully indexing according the observed past aggregate inflation rate. Therefore, when setting the optimal price at period \(t\) the firms take into account that in any future period \(j\) there is a probability \(\alpha_j\) that the price is the price at \(t\) plus full indexation until period \(j\). In other words, the price at \(t\), \(P_t(i)\) has a probability \(\alpha_j\) of surviving until \(t + j\):

\[
P_{t+j}(i) = P_t(i)\pi_{t+1}\pi_{t+2}\ldots\pi_{t+j-1} \equiv P_t(i)\Psi_{t,j},
\]

where \(\Psi_{t,0} = 1\). Also, we obtain the following identity

\[
\frac{P_t(i)\Psi_{t,j}}{P_{t+j}} = \frac{P_t(i)}{P_t} \frac{\pi_t}{\pi_{t+j}}.
\]

The firms pricing problem can be written as

\[
\max_{P_t(i)} \sum_{j=0}^{\infty} \alpha^j \Lambda_{t,t+j} \left\{ \frac{P_t(i)\Psi_{t,j}}{P_{t+j}} - mc_{t+j}(i) \right\} Q_{t+j}(i)
\]

subject to

\[
Q_{t+j}(i) = Q_{t+j} \left( \frac{P_t(i)\Psi_{t,j}}{P_{t+j}} \right)^{-\theta}
\]

where

\[
\Lambda_{t,t+j} = \beta^j \frac{\bar{c}_{t+j}}{C_t^{1-\sigma}}
\]

By using the first order conditions of the household maximization problem:

\[
\Lambda_{t,t+j} = \beta^j \frac{\lambda_{t+j}\Phi_M(1 + i_{t+j})}{\lambda_t\Phi_M(1 + i_t)} \equiv \beta^j \frac{\lambda_{t+j}}{\lambda_t}.
\]

The first order condition reads as follows:

\[
0 = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \bar{\Lambda}_{t+j} Q_{t+j}(\pi_{t+j}) \theta \left\{ \frac{\bar{P}_t}{P_t} \frac{\pi_t}{\pi_{t+j}} - frac{\theta}{\theta} - 1mc_{t+j} \right\}.
\]
where $\tilde{P}_t$ is the optimal price.

From (35) and (52) we can get the law of motion of the aggregate price index:

$$P_t^{1-\theta} = \alpha(P_{t-1}\pi_{t-1})^{1-\theta} + (1-\alpha)\tilde{P}_t^{1-\theta}$$

(59)

and dividing this equation by $P_t^{1-\theta}$ we get:

$$\tilde{p}_t\pi_t = \left(\frac{(\pi_t)^{1-\theta} - \alpha(\pi_{t-1})^{1-\theta}}{1-\alpha}\right)^{\frac{1}{\theta}} \equiv G(\pi_t, \pi_{t-1})$$

(60)

where $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$.

By substituting (60) into (58) we get the equation that determines the dynamic of inflation:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j N_{t+j}Q_t^{\pi_{t+j}} \theta \left\{ \frac{G(\pi_t, \pi_{t-1})}{\pi_{t+j}} - frac{\theta}{\theta - 1}mc_{t+j} \right\}.$$  

(61)

### 2.2.4 Domestic Exporters

We think of exporters as competitive firms producing a single agricultural commodity that is sold in international markets. They are price takers in both the intermediate input market and in the international market in which they sell their output. In order to produce they use domestic goods according to the following production function:

$$X_t = (Q_t^{DX})^{\alpha_A}$$

(62)

where $Q^{DX}$ is the quantity of inputs used by this sector and $\alpha < 1$, i.e. firms that operate in this sector have decreasing returns of scale which stands for the use of land (which is not modeled), a fixed factor of production.

These firms maximize the following profit function

$$\Pi_t^X = S_t^i P_t^{X_t} X_t - P_t Q_t^{DX}$$

(63)

subject to (62).

By solving the problem we get

$$Q_t^{DX} = \alpha_A (e_t P_t^*) X_t = (\alpha_A e_t P_t^*) \frac{1}{1-\alpha_A}$$

(64)

which is the factor demand function in which $e_t = \frac{S_t^i P_t^{X_t}}{P_t}$ and $P_t^* = \frac{p_t^{X_t}}{P_t}$ represent the real exchange rate and external terms of trade of this small open economy, respectively.
The external terms of trade are assumed to be exogenous and so completely determined in the last rest of the world. By inserting (65) into (62) we get the expression for total exports in equilibrium which reads:

\[ X_t = (\alpha_A e_t P^*_t)^{\frac{\alpha_A}{1-\alpha_A}} \]  

(65)

which, as can be seen, depends on the product of the real exchange rate and the external terms of trade.

### 2.2.5 Final Goods Importers

There are firms operating in perfectly competitive markets that combine imported intermediate inputs varieties, \( N_t(i) \), according to a CES production function in order to produce a final imported good \( N_t \). The production function of the representative firm in this sector is

\[ N_t = \left( \int_0^1 N_t(i) \frac{\theta_N-1}{\theta_N} \, di \right)^{\frac{\theta_N}{\theta_N-1}}, \]  

(66)

with \( \theta_N > 1 \) being the elasticity of substitution between varieties of intermediate imported consumption goods.

As in the case of the domestic final good sector we can set up the firm’s maximization problem so we get the following demand function for each variety of intermediate imported good:

\[ N_t(i) = N_t \left( \frac{P_t^N(i)}{P_t^N} \right)^{-\theta_N}, \]  

(67)

being the resulting price index for imported goods given by the following expression

\[ P_t^N = \left( \int_0^1 P_t^N(i)^{1-\theta_N} \, di \right)^{\frac{1}{1-\theta_N}}, \]  

(68)

and the import cost bill

\[ P_t^N N_t = \int_0^1 P_t^N(i)N_t(i) \, di. \]  

(69)

### 2.2.6 Intermediate Goods Importers

There is a continuum of monopolistically competitive firms that buy a bundled final good abroad at a foreign currency price \( P^{\times N} \) and convert it into a differentiated good that is sold in the domestic market in domestic currency. The purchase price in domestic currency is \( S_t P^{\times N} \) where \( S_t \)
is the nominal exchange rate.

They maximize profits by choosing \( P^N_t(i) \) and taking the overall imported quantity \( N_t \) and price \( P^N_t \) as given and the final demand of good \( i \) given by equation (67). Therefore, they maximize the following profit function

\[
\Pi^N_t = N_t(i) \{ P^N_t(i) - S_t P^*N_t \} = N_t \left( P^N_t \right)^{-\theta_N} \left\{ \left( P^N_t(i) \right)^{1-\theta_N} - S_t P^*N_t \left( P^N_t(i) \right)^{-\theta_N} \right\}
\]  
(70)

The resulting first order condition is

\[
N_t \left( P^N_t \right)^{-\theta_N} \left( P^N_t(i) \right)^{-\theta_N-1} (1-\theta_N) \left\{ P^N_t(i) - \frac{\theta_N}{\theta_N-1} S_t P^*N_t \left( P^N_t(i) \right)^{-\theta_N} \right\} = 0
\]  
(71)

The first order condition yields the optimal price:

\[
P^N_t(i) = \frac{\theta_N}{\theta_N-1} S_t P^*N_t.
\]  
(72)

By using (68) to aggregate and dividing by \( P_t \) we obtain an expression for the relative price of imported goods:

\[
p^N_t(i) = \frac{\theta_N}{\theta_N-1} e_t
\]  
(73)

which is a mark-up over the real exchange rate \( e_t \).

### 2.2.7 Banks

There is a competitive bank industry which are owned by households. They obtain one period funds, \( B^*B_t \), in international markets in which they are price takers and from households in the form of one period deposits \( D_t \). They use the funding to lend to firms in one period loans denoted by \( L_t \), lend or borrow in the interbank system (which is zero in equilibrium), and also trade short term central bank bonds \( B^{CB}_t \). The balance sheet constraint for the representative bank reads

\[
L_t + B^{CB}_t + \gamma^R D_t = D_t + S_t B^*B_t.
\]  
(74)

where \( \gamma^R \) represents the reserves requirement of bank deposits.

It is assumed that deposits and central bank bonds are perfect substitutes and so they pay the same interest rate \( i_t \), but we further assume that households cannot buy central bank bonds directly for some friction that it is not specified. Note that since banks operate exclusively in
domestic currency but they get part of their funding in international markets, they are exposed to exchange risk for which we assume they cannot insure against.

Specifically, for every unit of foreign currency they repay on period ahead they must have local currency in the amount

$$E_t \delta_{t+1} (1 + i^B_t),$$  \hspace{1cm} (75)

where $i^B_t$ is the nominal foreign interest rate and $\delta_{t+1}$ the depreciation rate which is:

$$\delta_{t+1} = \frac{S_{t+1}}{S_t}. \hspace{1cm} (76)$$

The interest rate at which the banks pay their foreign is a composite of the risk-free international interest rate, $i^*_t$, exogenously given and, a risk premium. As it is common in the literature, the risk premium depends on a stochastic exogenous component $\phi^*_B$ and an endogenous component that is an increasing and convex function of the bank’s foreign debt. In mathematical notation it is given by

$$1 + i^B_t = (1 + i^*_t) \left[ 1 + \phi^*_B + P_B \left( \frac{S_t B^*_B}{P_t} \right) \right], \hspace{1cm} (77)$$

where $P_B$ is an increasing an convex function.

We also assume that banks have a quadratic cost function that depends on the amount of the real loans conceded in the period before,

$$C^B_{t+1} = \frac{1}{2} a^B_L \left( \frac{L_t}{P_t} \right)^2 \hspace{1cm} (78)$$

with the parameter $a^B_L > 0$.

Therefore, each period the bank’s profit function is given by:

$$E_t \Pi^B_{t+1} = i^*_t L_t + i_t (B^CB - D_t) - E_t \delta_{t+1} i^B_t S_t B^*_B - P_t \frac{1}{2} a^B_L \left( \frac{L_t}{P_t} \right)^2. \hspace{1cm} (79)$$

The representative bank the maximizes (79) subject to (??bcB) and (??iB).

By solving this problem we get the supply of loans and the amount of foreign funding:

$$\frac{L^S_t}{P_t} = \frac{1}{a^B_L} \left( i^*_t - \frac{i_t}{1 - \gamma^B} \right) \hspace{1cm} (80)$$

and

$$i_t = (1 - \gamma^B) \left[ (1 + i^*_t) \phi_B (e_t B^*_B) - 1 \right] E_t \delta_{t+1} \hspace{1cm} (81)$$

where $b^*_B \equiv \frac{B^*_B}{i^*_B}$ is the real debt of the bank. Note that the supply of loans is a function of
the the rate of the reserve requirements and the interest rate differential with optimal amount of foreign funding depending on the uncovered interest parity which is adjusted by risk which is given by the function \( \varphi_B(.) \) defined as:

\[
\varphi_B(e_t, b_t^*) \equiv 1 + \phi_t^* + p_B(e_t, b_t^*) + e_t b_t^* p'_B(e_t, b_t^*). \tag{82}
\]

Given \( L_t^S, D_t^S \) and \( B_t^* \), from (74) we get the demand for central bank bonds:

\[
B_{t^{CB,D}} = (1 - \gamma_R)D_t^S + S_t B_t^* - L_t^S. \tag{83}
\]

2.3 Public Sector

The public sector is composed by the central bank and the consolidated public sector or central government.

2.3.1 Central Bank

The Central Bank issues local currency \( M_t^0 \), debt certificates to banks for non-remunerated reserves \( R_t^B \), domestic bonds \( B_t^{CB} \) denominated in local currency which we assume are exclusively hold by the domestic banking sector. It holds international reserves \( R_t^{*CB} \) in the form of foreign currency risk-less bonds. The budget constraint of the monetary authority reads

\[
R_t^B + M_t^0 + B_t^{CB} - S_t R_t^{*CB} = M_{t-1}^0 + (1 + i_{t-1}) B_{t-1}^{CB} - (1 + i_{t-1}) S_{t-1} R_{t-1}^{*CB} + R_{t-1}^B = \left[ M_{t-1}^0 + R_{t-1}^B + B_{t-1}^{CB} - S_{t-1} R_{t-1}^{*CB} \right] - \left[ i_{t-1} S_t R_t^{*CB} + (S_t - S_{t-1}) R_{t-1}^{*CB} - i_{t-1} B_{t-1}^{CB} \right]. \tag{84}
\]

We also assume that the central bank’s balance sheet is always preserved by assuming the quasi-fiscal result (second term in brackets of equation (84)) is transferred to the central government in every period, therefore we have that

\[
M_t^0 + R_t^B = S_t R_t^{*CB} - B_t^{CB}, \tag{85}
\]

which means that the central bank satisfies household demand for currency by issuing bonds or intervening in the foreign currency market.

2.3.2 Monetary Policy

Providing the similarities in the observed monetary policy regime followed, as in the case of the model applied to the Argentine economy, it is assumed that there is an Inflation Targeting with
managed exchanged rate floating regime. The Central Bank intervenes in the foreign exchange markets as well as in the money market aiming to an operational target for the interbank interest rate \( i_t \) and at the same time to attenuate real currency depreciations or appreciations, \( e_t/e_{t-1} \). For these purposes the monetary authority follows two simple monetary policy feedback rules.

The first one in one where the central bank responds to deviations of the consumption inflation rate (or CPI inflation rate) from the target, \( \pi_T^C \), and to deviations of the GDP and the real exchange rate from its steady states values (non-stochastic). The interest rate feedback rule includes its lagged value and reads as follows

\[
1 + i_t = (\Xi^{TR})^{1-h_0} (1+i_{t-1})^{h_0} \left( \frac{\pi_T^C}{\pi_T} \right)^{h_1} \left( \frac{Y_{t-1}}{Y} \right)^{h_2} \left( \frac{e_t}{e} \right)^{h_3}
\]

with \( h_0 \geq 0, h_1 > 0, h_2 \geq 0, h_3 \geq 0 \) and, where

\[
\Xi^{TR} = \left( \frac{\pi}{\pi^C} \right)^{h_0} (1+i).
\]

The variables without time period subscripts represent the steady state values.

As mentioned, the Central Bank also intervenes in the foreign exchange market. It is assumed that the monetary authority has a long run operational target level of international reserves, \( \gamma^T \). The Central Bank has the following feedback rule in which it intervenes against real appreciations or depreciations, has a preferences for smoothing the variations in international reserves and as mentioned, has a long run target for its international reserves:

\[
r^{*\text{CB}} = (\gamma^T)^{1-k_0} \left( r^{*\text{CB}}_{t-1} \right)^{k_0} \left( \frac{e_t}{e_{t-1}} \right)^{-k_1}
\]

with \( k_0 \in (0,1) \) and \( k_1 > 0 \) and where real reserves are defined as \( r^{*\text{CB}} \equiv \frac{R^{*\text{CB}}}{P^{*\text{NP}}} \).

### 2.3.3 Central Government

We assume that the government runs a balanced budget every period. The fiscal policy is simplified by assuming that there is an exogenous stream of real government expenditures \( G_t \) which can be financed by the Central Bank’s result and lump sum taxes \( T_t \). Therefore, the central government budget constraint is:

\[
T_t = P_t G_t - \left\{ i_{t-1}^* + (1 - S_{t-1}/S_t) \right\} S_t R^{*\text{CB}}_{t-1} - i_{t-1} B^{*\text{CB}}_{t-1} \right\}. \quad (89)
\]
3 Equilibrium

3.1 Market Clearing Conditions

In the labor market, the households aggregate labor supply $h_t$ equals firms labor demand given by equation (48):

$$h_t = \frac{1}{\epsilon_t \kappa} b^q \left( \frac{p^N_t}{w_t} \right)^{(1-b^q)} Q_t.$$  (90)

Regarding the clearing of the loan market, the loan supply (equation (80)) equals firm’s loan demand (equation (51)) and that gives the loan interest rate in equilibrium:

$$i^L_t = \frac{i_t}{(1-\gamma^R)} + \frac{a^B}{b^q} E_t \left( w_{t+1} h_{t+1} \right).$$  (91)

In the market for domestic goods, the output produced by domestic firms, $Q_t$, must equal the demand for these goods from households, Government and from domestic exporters $^1$:

$$Q_t = a_D p^C_t C_t + G_t + (\alpha_A e_t p^*_{t})^{\frac{1}{1-\alpha_A}}.$$  (92)

The market for imported goods also clears, i.e. total imports equals the demand for imported goods done by households and firms:

$$p^N_t N_t = (1 - a_D) p^C_t C_t + p^N_t N^D_t.$$  (93)

and, by using equation (50) we get

$$p^N_t N_t = (1 - a_D) p^C_t C_t + \frac{1 - b^q}{b^q} w_t h_t.$$  (94)

We now define GDP in terms of the model, which is the sum of household and government consumption plus exports minus imports and reads:

$$Y_t = p^C_t C_t + G_t + (\alpha_A e_t p^*_{t})^{\frac{a_A}{1-\alpha_A}} - p^N_t N_t.$$  (95)

and, by using equations (94) and (14):

$$Y_t = a_D p^C_t C_t + G_t + (\alpha_A e_t p^*_{t})^{\frac{a_A}{1-\alpha_A}} - \frac{1 - b^q}{b^q} w_t h_t.$$  (96)

$^1$Note that the transactions costs also consume resources as well as the banking activity. However, following Escude (2007), for simplification purposes we abstract from this in this version of the model.
Therefore, combining (92) and (96) we can obtain the expression that relates output and GDP:

\[ Y_t = Q_t - \frac{1 - b^\eta}{b^\eta} w_t h_t + (\alpha_A e_t p_t^*)^{\frac{\alpha_A}{1-\alpha_A}} - (\alpha_A e_t p_t^*)^{\frac{1}{1-\alpha_A}}. \]  

(97)

Finally, the expression for the balance of payments reads as follows,

\[ B_t^{*B} - R_t^{*CB} = (1 + i_t B_{t-1})B_t^{*B} - (1 + i_t^{*})R_{t-1}^{*B} - \left[ (\alpha_A e_t p_t^*)^{\frac{\alpha_A}{1-\alpha_A}} - (1 - \alpha_D) p_t^C C_t - \frac{1 - b^\eta}{b^\eta} w_t h_t \right], \]  

(98)

where the term in brackets is the trade balance.

**The system**  We can now summarize the system of equations that compose the model where all variables are expressed in real terms.

The dynamic of consumption is given by equation (30). The real wage by (31). We also have the domestic inflation equation (Phillips curve) given by (61).

Additionally, we have the inflation rate for consumption goods in equation (17). Imported goods price (73) and the two related identities \( \frac{p_t^N}{p_t^{*}} = \pi_t^N \) and \( \frac{e_t}{e_t^{*}} = \delta_{t} \pi_t^N / \pi_t^{*} \). The balance of payments given by (98). The uncovered interest rate parity (81). The real marginal cost (47).

The loans, labor and domestic goods markets clearing conditions given by (91), (90) and (92), respectively.

We also have the expressions for the real GDP given by (97) and the relative price of consumption goods in (13).

Finally we have the interest rate rule given by equation (86) and the equation that determines the rule for foreign exchange market interventions (88).

Therefore, we have 17 equations and 17 endogenous variables which are: \( i_t, i_t^{*}, \pi_t, \delta_t, \pi_t^N, \pi_t^C, p_t^N, w_t, e_t, m_{t}, p_t^C, C_t, h_t, Q_t, Y_t, R_t^{*CB}, b_t^{*B} \).

In addition, after solving for these variables, by using the model relationships we can solve for other variables such as the stock of money \( m_t^0 \), the supply of loans in real terms \( l_t \), the stock of Central Bank’s bonds \( b_t^{CB} \), the real tax revenues \( t_t \), etc.

**Auxiliary Functions**  Before analyzing the steady state equilibrium we need to specify some exogenous auxiliary functions.

The first one is the transaction cost function which we assume takes the following form:

\[ \tau_M(\omega_t) \equiv a_M \omega_t + \omega_t^{-b_M} - c_M \] 

(99)
with \( a_M, b_M, c_M > 0 \).

Combining (99) with (24) we get the liquidity preference function:

\[
\omega_t = \frac{m^0_t}{P_t^c c_t} = \xi(1 + i_t) = \left[ \frac{b_M}{a_M + 1 - \left( \frac{1}{1 + i_t} \right)} \right]^{\frac{1}{1+b_M}}.
\]

(100)

As a result, the elasticity of the demand for money with respect to the gross interest rate reads as follows,

\[
\varepsilon^{m0}_t = \frac{\omega_t^{1+b_M}}{(1+b_M)b_M(1+i_t)}.
\]

(101)

Therefore, the auxiliary function for the effect of expenditure on an increase in consumption, defined by (22) is now given by:

\[
\phi_M(\omega_t) = 1 - c_M + (1 + b_M)\omega_t^{-b_M}.
\]

(102)

Finally, the risk premium takes the following functional form:

\[
p_B(e_t b_t^*B) \equiv \alpha^{RP}_1 (e_t b_t^*B) \alpha^{RP}_2
\]

(103)

with \( \alpha^{RP}_1 > 0 \) and \( \alpha^{RP}_2 > 0 \).

Therefore, the function \( \phi_B(.) \) defined in (82) takes the form:

\[
\phi_B(e_t b_t^*B) = 1 + \phi_t^*B + (\alpha^{RP}_2 + 1)\alpha^{RP}_1 (e_t b_t^*B) \alpha^{RP}_2.
\]

(104)

4 Steady State Analysis

4.1 The system in the Non-stochastic Steady State

In this section we analyze the economy in its non-stochastic steady state around which we make log-linear approximations to the dynamic system. We express all the model equations with variables in their steady state values (without time subscripts). We first assume that in the steady state shocks \( \xi^C = \varepsilon = 1 \). In what follows I present the 17 equations that conform the model economy in its steady state.

The dynamic of consumption reduces to

\[
\beta (1 + i) = \pi.
\]

(105)

The real wage is given by
\[ w = \frac{\psi}{\psi - 1} h^\kappa C^\sigma \tilde{\varphi}_M (1 + i). \]  

(106)

The domestic inflation Phillips curve reads

\[ 1 = \frac{\theta}{\theta - 1} mc. \]  

(107)

The relative price of imported goods is now

\[ p^N = \frac{\theta_N}{\theta_N - 1} e. \]  

(108)

Then we have the two identities:

The relative price of imported goods is now

\[ 1 = \frac{\pi^N}{\pi}, \]  

(109)

and

\[ 1 = \frac{\delta \pi^{\pi^N}}{\pi}. \]  

(110)

The balance of payments reads as follows

\[ b^*B \left[ \frac{(1 + i^*)}{\pi^N} \left[ 1 + \phi^*B + p_B(e b^*B) \right] - 1 \right] - r^*CB \left[ \frac{(1 + i^*)}{\pi^N} - 1 \right] = \]  

\[ \left[ (\alpha_A e p^*) \right]^{\frac{\alpha_A}{\gamma - \alpha_A}} - (1 - a_D) p^C - \frac{1 - b^g}{b^g} \right] wh. \]  

(111)

The risk adjusted uncovered interest rate parity:

\[ i = [(1 + i^*) \varphi B (e b^*B) - 1] \delta. \]  

(112)

From the loan market clearing condition we get the interest rate on loans in steady state:

\[ i^L = \frac{i}{(1 - \gamma^R)} + \frac{a^B \zeta}{b^g} \omega, \]  

(113)

and from the labor market clearing condition:

\[ h = \frac{b^g}{\kappa} \left( \frac{p^N}{w} \right)^{1-b^g} Q. \]  

(114)
We also have the equation from the domestic goods market clearing condition

\[ Q = a_D p^C C + G + (\alpha_A e p^*)^{\frac{1}{1-\alpha_A}}, \]  
(115)

the expression obtained for the GDP in its steady state value

\[ Y = Q - \frac{1-b_g}{b_g} wh + (\alpha_A e p^*)^{\frac{\alpha_A}{1-\alpha_A}} - (\alpha_A e p^*)^{\frac{1}{1-\alpha_A}}, \]  
(116)

and the real marginal cost,

\[ mc = \frac{1}{\kappa} (1 + \zeta L) w^{b_g} (p^N)^{1-b_g}. \]  
(117)

The relative price of consumption is given by

\[ (p^C)^{1-\theta_C} = a_D + a_N (p^N)^{1-\theta_C}, \]  
(118)

and the inflation rate of consumption goods

\[ (\pi^C)^{1-\theta_C} = \frac{a_D}{a_D + a_N (p^N)^{1-\theta_C}} (\pi)^{1-\theta_C} + \left( 1 - \frac{a_D}{a_D + a_N (p^N)^{1-\theta_C}} \right) (\pi^N)^{1-\theta_C}. \]  
(119)

Then we have the two policy rules, the interest rate rule given by

\[ (1+i)^{1-h_0} = (\Xi^{TR})^{1-h_0} \left( \frac{\pi^C}{\pi^T} \right)^{h_1}, \]  
(120)

with

\[ \Xi^{TR} = \left( \frac{\pi}{\pi^C} \right)^{h_1} (1+i), \]  
(121)

and foreign exchange market intervention rule,

\[ (r^{*CB})^{1-k_0} = (\gamma^T)^{1-k_0}, \]  
(122)

5 The dynamic system

In the Appendix I proceed to develop the log-linear approximation of the system of equations detailed above and to reduce it to write it un matrix form. The system get reduced to 6 equations: the Taylor rule equation (167), the equation of foreign market interventions (168), the balance of payments (204), the IS (215), the domestic inflation Phillips equation (216), the risk adjusted uncovered interest rate parity (225).
6 Empirical Analysis

6.1 Calibration

We first proceed to calibrate the model to the non-stochastic steady state. The strategy followed is to set parameters values and steady state values for some of the endogenous variables that are consistent with the Uruguayan economy in 2005. Having done this we use the relations described in sections 7.2 and 7.3 to obtain values for the coefficients of the equations that form the dynamic system. With these inputs we can then simulate the economy in order to obtain impulse-response functions.

I start by fixing the values of variables corresponding to the rest of the world. I set $\pi^* = 1.006$ which is the US CPI inflation rate in the period considered and, $i^* = 1.03265$ that corresponds to the 3 months US Treasury Bills in the period. These correspond to a real annual risk-free interest rate of $(1 + i^*)/\pi^* = 1.0265$.

In addition, we assume that $\beta = 0.996$, and in line with the previous literature the utility function parameters take the following values: $\chi = 1$ and $\sigma = 1$.

I also assume that the level of GDP in SS is the value average of the period 1994-2005 in millions of uruguayan 2005 constant pesos, i.e. $Y = 431.8$.

In order to have values for the assets and liabilities of the banking system we need to use the balance sheets of the central bank and the banking system.

From the central bank’s balance sheet we have that on average, $m^0/Y = 0.04$ and international reserves are 15% of GDP, therefore, by using equation 85 (divided by $Y$) we have

$$0.04 = 0.15 - 0.11.$$  

In addition, using data from the balance sheet of the banking sector we have that $l/Y = 0.35$. From Escude (2007) I take the values for the parameters of the bank’s risk premium function, i.e. $\alpha_1^{RP} = 0.0037$ and $\alpha_2^{RP} = 0.099$. Assuming we have a zero trade balance (recall we the model does not consider foreign public debt), given the values for these parameters we need to assume that $eb^*B/Y = 0.09$ (see equation (111)). Therefore, taking equation 74 (divided by $Y$) we have that

$$0.35 + 0.11 = 0.09 + 0.37$$

which means that the total deposits in the banking sector are 37% of GDP in steady state.\footnote{In the period analyzed the average value for deposits is 53%, however this number is probably affected by the fact that the uruguayan financial sector used to play a major role as a safe regional alternative for foreign depositors. This changed after the 2002 crisis and now the number is closer to 40%.}
Regarding the target for the inflation rate I choose \( \pi^T = 0.06 \) which is the upperbound of the target range for the inflation rate announced by the authorities from June, 2011.

Given that \( \pi^T = 0.06 \), by using (105) and (110) we get that \( i = 0.0643 \) and \( \delta = 1.0537 \).

Therefore, we have that the value for the gross uncovered interest rate parity risk premium is:

\[
\left[ 1 + \phi^*b + (\alpha_2^{RP} + 1)\alpha_1^{RP} (eb^*B)^{\alpha_2^{RP}} \right] = 1.0275.
\]

In addition, according to the data, imports are around 28% of GDP, i.e. \( p^N N / Y = 0.28 \) and \( G / Y = 0.114 \). Hence, given that we have that \( TB / Y = 0 \), it must be the case that \( X / Y = 0.28 \) and using the accounting identity we have that

\[
p^C C / Y = 1 - G / Y - X / Y + p^N N / Y = 1 - 0.11 = 0.89
\]

Given this number and taking into account that \( m^0 / Y = 0.04 \) we have that \( \omega = 0.04 / 0.89 = 0.0449 \).

Furthermore, according to the data around 60% of imports are inputs, which means \( \frac{p^N N D}{p^N N} = 0.60 \). Therefore, the rest, 40% is for private consumption. Providing \( p^N N / Y = 0.28 \), then the share of imports in domestic consumption is \( a_N = \frac{0.28 \times 0.40}{0.89} = 0.1258 \) and so \( a_D = 1 - 0.1258 = 0.8742 \) and, the ratio of imported inputs over GDP is \( \frac{p^N N D}{Y} = 0.60 / 0.28 = 0.168 \).

We now proceed to find values for the parameters of the liquidity preference function. Recall that given our functional form assumption (equation (100)) we derive an expression for the elasticity of the cash demand with respect to the interest rate which was given by equation (101). According to Brum, et. al. (2011), the long run interest rate elasticity of private demand for cash is 0.30, so by using equation (101) together with the values obtained for \( \omega \) and \( i \), we have that \( b_M = 0.0955 \). With this value for \( b_M \) we are now ready to get \( a_M \) by just using equation (100), so we obtain \( a_M = 2.7989 \). Finally, we still need to get a value for \( c_M \).

After assuming that the transaction cost in terms of domestic goods represent 1% of private consumption, i.e. \( \tau_M = 0.1 \), then by using equation (99) we get \( c_M = 1.4606 \). These values implies that \( \varphi_M (\omega) = 1.01128 \) (use equation (102)).

I now turn to calibrate the parameters of the loan supply function. Recall that we have \( L / Y = 0.35 \). In addition, according to the data on active interest rate from 2005, its average value is \( i^L = 0.125 \). Finally, in 2005 the mean of banks reserve requirements was 0.15. Therefore, by using equation (80), we have that \( a^B_L = 0.1331 \).

To calibrate the domestic sector production function we make use of equation ((50)) which
is then divided by $Y$. In addition, we already have that $p^N N^D / Y = 0.168$, therefore

$$\frac{wh}{Y} = \frac{b^q}{1 - b^q} 0.168.$$  

After assuming that the share of labor inputs in production is 0.7 (the left hand side of the equation) we obtain the parameter which is $b^q = 0.8064$.

Since we know have $b^q$ we are ready to obtain the value of the parameter $\varsigma$ which represents the fraction of firm’s funding that is provided by the banking system. Using equation (51) and the obtained values of the variables involved in it, we have that $\varsigma = 0.4032$

By using information from input-output tables provided by the central bank, I set the demand of domestic inputs by the primary producing sector to 5.2% of GDP in 2005. So by using equation (65) we have that:

$$\frac{Q^{DX}}{Y} = \frac{(\alpha A e p^*)^{\frac{1}{1 - \alpha_A}}}{Y} = 0.052.$$  

Additionally, by using equation (115) we obtain the ratio of output over GDP:

$$\frac{Q}{Y} = a_D \frac{p^C C}{Y} + \frac{G}{Y} + \frac{Q^{DX}}{Y} = 0.94.$$  

We are now ready to obtain $\alpha_A$ by using (65) as follows:

$$0.052 = \alpha_A e \frac{p^* X}{Y} = \alpha_A \times 0.28,$$

so $\alpha_A = 0.1857$.

Again, by using (65):

$$0.052 = \frac{(\alpha A e p^*)^{\frac{1}{1 - \alpha_A}}}{Y}$$

We still need to calculate the marginal cost, $mc$. For this matter, we use equation (46) as follows:

$$mc = \frac{(1 + \varsigma i^L) \left(\frac{wh}{Y} + \frac{p^N N^D}{Y}\right)}{\frac{Q}{Y}} = 0.969.$$  

In addition, by using equation (65) we have that $\theta = 32.25$.

Recall that importers charge a markup of $\frac{\theta_N}{1 - \theta_N}$ over real exchange rate (see equation (73)). We assume that $\theta_N = 3.5$ which implies a markup of 40%.

Regarding the elasticity of substitution between imported and domestic consumption goods (also called Armington elasticity), there is a vast literature (see for instance Rhul (2008) and Balistreri and McDaniel (2002)) which finds that its value is in the range of 1.2 and 6.4. I take
the average of these two values, so I assume $\theta_C = 3.8$.

Now, using equation (165) we have that

$$a_{PC} = \frac{(1 - aD) \left( \frac{\theta_N}{\theta_N - 1} e \right)^{1 - \theta_C}}{aD + (1 - aD) \left( \frac{\theta_N}{\theta_N - 1} e \right)^{1 - \theta_C}} = \frac{(1 - 0.8742)(1.4 \times e)^{(1-3.8)}}{0.8742 + (1 - 0.8742)(1.4 \times e)^{(1-3.8)}} = 0.10$$

where the value of 0.10 for $a_{PC}$ was taken from Gianelli (2011). Therefore, we have an equation in $e$ and after solving it we have that $e = 0.7834$. Providing $e \times p^* = 67.849$ we have that $p^* = 86.61$.

Regarding the stochastic processes specified for the shocks we set the following values for persistent parameters: $\rho_e = 0.1, \rho_G = 0.77, \rho_{\varepsilon} = 0.87, \rho_{\varepsilon^*} = 0.6, \rho_{p^*} = 0.9, \rho_{\pi^*} = 0.2, \rho_{\pi^\pi} = 0.7$ and $\rho_{p^*} = 0.3$.

Finally, we need to set values for the parameters of the policy rules functions. For this matter I follow Escude (2007) and set $h_0 = 0.5, h_1 = 1.5, h_2 = 1, h_3 = 1, k_0 = 0.1$ and $k_1 = 1$.

We now have parameterized the system and so by using these values we are ready to find the values for the whole set of coefficients of the system of linearized equations, explained in detail in the previous section. Once we find values for the coefficients we are ready to simulate the economy and obtain impulse response functions after hitting the economy with shocks to the exogenous variables.

### 6.2 Impulse Response Analysis: The Benchmark Case

In this section we use the model to analyze impulse response functions derived from the computation of the model for the benchmark case. This case is defined as the one in which the reserve requirement rate is in its 2005 value, that means, $\gamma^R = 0.15$.

Specifically, we analyze the behavior of a set of macroeconomic variables when the economy is in its steady state and is hit by one of the exogenous shocks that were introduced in the model: productivity shock, international interest rate shock, government expenditure shock, international inflation rate, demand shock, international export prices and monetary policy shock. We hit the economy with shocks of one standard deviation of each of the exogenous variables at a time. Specifically, we are interested in the reaction of the interest rate ($i$), international reserves ($res$), foreign debt ($b^B$), GDP ($y$), inflation rate ($pi$) and exchange rate ($e$) when the economy is hit by the different shocks considered.

Figure 7 corresponds to the reaction of this variables to a shock to government expenditures. Figure 7 depicts the behavior of these variables when the model economy is hit by a positive
shock of international export prices. In figure 7 I show the evolution of these variables to a international inflation rate shock.

In figure 7 we show the evolution of the variable of interest when the economy is hit by a productivity shock. Figure 7 when it is hit by a shock to international interest rate. In addition, in figure 7 we show the response to a positive shock to the interest rate risk premium. In figure 7 the economy is hit by a monetary policy shock modeled as an increase of foreign currency purchases by the central bank. Finally, figure 7 depicts the behavior of the economy when it is hit by a demand shock.

The results are similar the findings of Cubas (2011). Although the reserve requirements were absent in his environment the inclusion of this monetary policy instrument does not imply major changes in the reaction of macroeconomic variables to the shocks considered. As we will see below, only substantial changes in the rate of reserves requirements affect real variables, but not the mean rates around 15% which is the one that corresponds to the benchmark case.

6.3 Changes in the Reserve Requirement Rate

In this section we use the model to analyze the case of an increase in the rate of bank reserves requirement. Specifically, we compute the impulse response functions when $\gamma^R = 0.22$ which is the maximum mean rate observed in Uruguay in the period 2005-2012. This rate correspond to the mean rate observed between June, 2008 and September, 2009 when the reserves requirement rate of short run deposits (less than 30 days) was increased from 17% to 25%. Figures 7 through 7 show the results. As when we compare the non reserve requirement case (Cubas (2011)) and the benchmark case, there are not substantial effects in the reaction of macroeconomic variables to the shocks considered. That means, eve though we increase the rate of reserve requirements from 15% to 22% we do not observe much action in our variables of interest.

Interestingly, we start to observe different responses when we rise $\gamma^R$ to 0.5. In this experiment we recompute the model by using this hypothetical rate which can be considered as an upperbound for this instrument. Figures 7 through 7 show the results. It is worth to notice that compared to the benchmark case, the main effects are when the economy is hit by a shock to government expenditures, a monetary shock or a shock to the international interest rate or to the interest rate premium.

For the case of a shock to government expenditures, by comparing Figures and 7 and 7 we notice the different reaction of the interest rate and the risk premium at the moment of the shock, both rise more for the case of $\gamma^R = 0.5$. In the event the economy is hit by a monetary shock, when the reserve requirement is low, at the moment of the shock the exchange rate reacts much
more than in the case of a high rate of reserve requirement. At the same time GDP decreases more and the reaction of inflation rate is also lower. In all cases the pattern to converge to the steady state equilibrium is similar with the exception of GDP which is more volatile for a higher $\gamma^R$. When the economy is hit by both a shock to the interest rate premium and the international interest rate we observe that in the case of a high $\gamma^R$ there is a stronger reaction of the inflation rate both at the moment of the shock and at the peak of the reaction. Also, with both shocks output is more volatile.

7 Concluding Remarks

This paper proposes a general equilibrium model for the Uruguayan economy designed to analyze the role of the reserve requirement rate as tool for monetary policy analysis. For this purpose we model a central bank that closely mimics the type of policy carried out by the Central Bank of Uruguay: inflation targeting policy and interventions in the foreign exchange market. The model is calibrated to the Uruguayan economy in 2005. We then compute impulse response functions for different reserve requirements regimes, in particular: comparing the benchmark case when the rate of reserve requirements is 15% with to cases: when the rate is 22% (the maximum observed in Uruguay in the period 2005-2011) and when it is 50% which is considered an upper bound. Interestingly, we only observe substantial changes in the variables, mainly the inflation rate and output, when the rate increases from 15% to 50% but only minor changes in the more realistic cases, that means, when the rate increases from 15% to 22%.
References


Figures

Figure 1: Shock to Government Expenditures
Figure 2: Shock to international prices
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Figure 23: Demand Shock
Appendix

7.1 Log-linearization

I first present the log-linearized equations in the order they appear in the steady state analysis in which we have changed the notation so that variables are expressed as deviations from their steady states values.

The dynamic of consumption is given by

\[ \hat{C}_t = E_t \hat{C}_{t+1} - \frac{1}{\sigma} \left( \hat{i}_t - E_t \hat{\pi}_{t+1} \right) + \frac{\varepsilon_M}{\sigma} E_t \Delta \hat{C}_{t+1}. \]  

(123)

where \( \varepsilon_M = \frac{b_M(1+b_M)\omega^{-bM}}{1-c_M+(1+b_M)\omega^{-bM}} \) (the elasticity of the auxiliary function with respect to the gross domestic interest rate) with \( \alpha_{CM}^{CD} = \frac{1}{(b_M+1)(\alpha M+1)(1+i)} \) (the elasticity of the cash household consumption with respect to the gross domestic interest rate).

The real wage is given by

\[ \hat{w} = \sigma \hat{C}_t + \varepsilon_M \hat{i}_t + \chi \hat{h}_t - \hat{C}. \]  

(124)

The domestic inflation Phillips curve reads

\[ \hat{\pi}_t - \hat{\pi}_{t-1} = \beta (E_t \hat{\pi}_{t+1} - \hat{\pi}_t) + \frac{(1-\alpha)(1-\alpha \beta)}{\alpha} \hat{m}_C_t. \]  

(125)

The relative price of imported goods is now

\[ \hat{p}^N_t = \hat{e}_t. \]  

(126)

Then we have the two identities:

\[ \hat{p}^N_t - \hat{p}^N_{t-1} = \hat{\pi}_t - \hat{\pi}_{t-1}, \]  

(127)

and

\[ \hat{\delta}_t = \hat{e}_t - \hat{e}_{t-1} + \hat{\pi}_t - \hat{\pi}^{*N}_t. \]  

(128)

The balance of payments reads as follows

\[ \gamma^{BP}_{3} \hat{b}_t^{*B} = \gamma^{BP}_{3} \hat{b}_{t-1}^{*B} + \gamma^{BP}_{1} \hat{r}_t^{*B} - \gamma^{BP}_{1} \hat{r}_{t-1}^{*B} + \gamma^{BP}_{4} \hat{e}_t + \gamma^{BP}_{5} \hat{e}_{t-1} + \gamma^{BP}_{6} \left( \hat{p}_t \hat{C} + \hat{C}_t \right) + \gamma^{BP}_{7} \left( \hat{w}_t + \hat{h}_t \right) - \gamma^{BP}_{2} \hat{i}_t^{*B} + \gamma^{BP}_{8} \hat{\phi}_t^{*B} + \gamma^{BP}_{2} \hat{\pi}_t^{*B} + \gamma^{BP}_{9} \hat{p}_t^{*B}. \]  

(129)
where

\[ \gamma_{BP}^1 = \frac{r^{*CB}}{r^{*CB} + \frac{1 + \pi \gamma_{N}}{\pi} \left[ 1 + \phi^{*B} + \alpha_{1}^{BP} (b^{*B}) \alpha_{2}^{BP} \right]}, \]  
(130)

\[ \gamma_{BP}^2 = \frac{TB}{TB + b^{*B} + \frac{1 + \pi \gamma_{N}}{\pi} r^{*CB}}, \]  
(131)

\[ \gamma_{BP}^3 = \frac{b^{*B}}{TB + b^{*B} + \frac{1 + \pi \gamma_{N}}{\pi} r^{*CB}}, \]  
(132)

with \( TB \) being the trade balance in the steady state. In addition we have that

\[ \gamma_{BP}^4 = 1 - \gamma_{BP}^2 - \gamma_{BP}^3, \]  
(133)

\[ \gamma_{BP}^5 = \gamma_{BP}^1 - \gamma_{BP}^3, \]  
(134)

\[ \gamma_{BP}^6 = \gamma_{BP}^1 \left[ (1 - \gamma_{BP}^1) \left[ (1 - \alpha_{2}^{RP}) \alpha_{2}^{BP} + 1 \right] \right], \]  
(135)

with \( \alpha^{RP} \) being the bank’s structure of foreign funding risk premium and given by

\[ \alpha^{RP} \equiv \frac{1 + \phi^{*B}}{1 + \phi^{*B} + \alpha_{1}^{RP} (b^{*B}) \alpha_{2}^{RP}}. \]  
(136)

\[ \gamma_{BP}^4 = \gamma_{BP}^2 \left( \frac{1}{\gamma^{TB}} \frac{\alpha_{2}}{1 - \alpha_{2}} - 1 \right), \]  
(137)

with \( \gamma^{TB} \) being the trade balance to export coefficient given by

\[ \gamma^{TB} \equiv \frac{eTB}{eTB + p\gamma_{N}}; \]  
(138)

\[ \gamma_{BP}^5 = (1 - \gamma_{BP}^1) (1 - \alpha_{2}^{RP}) \alpha_{2}^{BP}, \]  
(139)

\[ \gamma_{BP}^6 = \gamma_{BP}^2 \left( \frac{1}{\gamma^{TB}} \left( \frac{1}{1 - \alpha_{2}} \gamma_{N} \right) \right), \]  
(140)

with \( \gamma^{N} \) representing the structure of imports which is given by

\[ \gamma^{N} \equiv \frac{(1 - \alpha_{D}) p^{C} C}{(1 - \alpha_{D}) p^{C} C + p^{N} N}; \]  
(141)

\[ \gamma_{BP}^7 = \gamma_{BP}^2 \left( \frac{1}{\gamma^{TB}} - 1 \right) (1 - \gamma^{N}), \]  
(142)
\[
\gamma_8^{BP} = (1 - \gamma_1^{BP}) \alpha^{RP},
\]
(143)

\[
\gamma_9^{BP} = \gamma_2^{BP} \frac{1}{\alpha_A} \frac{\alpha_A}{1 - \alpha_A}.
\]
(144)

The risk adjusted uncovered interest rate parity:

\[
\hat{i} = \gamma_1^B \{ E_t \hat{\delta}_{t+1} + \gamma_2^B \left[ (1 - \alpha_{RP}) \alpha_2^{RP} (\hat{e}_t + \hat{b}_t^{*B}) + \alpha_{RP} \hat{\phi}_t^{*B} + \hat{\eta}_t^{*B} \right] \} \delta,
\]
(145)

where

\[
\bar{\alpha}^{RP} \equiv \frac{1 + \phi^{*B}}{1 + \phi^{*B} + \alpha_2^{RP} (b^{*B} e) \alpha_2^{RP}}.
\]
(146)

\[
\gamma_1^B \equiv \frac{\delta (1 + i^*) \left\{ 1 + \phi^{*B} + (\alpha_2^{RP} + 1) \alpha_1^{RP} (b^{*B} e) \alpha_2^{RP} \right\} - 1}{1 + \delta (1 + i^*) \left\{ 1 + \phi^{*B} + (\alpha_2^{RP} + 1) \alpha_1^{RP} (b^{*B} e) \alpha_2^{RP} \right\} - 1},
\]
(147)

and

\[
\gamma_2^B \equiv \frac{(1 + i^*) \left[ 1 + \phi^{*B} + (\alpha_2^{RP} + 1) \alpha_1^{RP} (b^{*B} e) \alpha_2^{RP} \right]}{(1 + i^*) \left[ 1 + \phi^{*B} + (\alpha_2^{RP} + 1) \alpha_1^{RP} (b^{*B} e) \alpha_2^{RP} \right] - 1}.
\]
(148)

The loan market clearing condition:

\[
\hat{\gamma}_t^{L} = \gamma_1^L \frac{1}{(1 - \gamma^R)} + \gamma_2^L E_t \left( \hat{w}_{t+1} \hat{\delta}_{t+1} \right),
\]
(149)

where

\[
\gamma_1^L \equiv \frac{1 + i^{L}}{1 + i^{L} \left( 1 - \gamma^R \right)},
\]
(150)

and

\[
\gamma_2^L \equiv \frac{i^{L} - \frac{i}{1 + \gamma^R}}{1 + i^{L}}.
\]
(151)

From the labor market clearing condition:

\[
\hat{h}_t = (1 - b^q) (\hat{p}_t^N - \hat{w}_t) + \hat{Q}_t - \hat{e}_t.
\]
(152)

We also have the equation from the domestic goods market clearing condition

\[
\hat{Q}_t = \gamma_1^Q \left( \hat{p}_t^C + \hat{C}_t \right) + \gamma_2^Q \frac{1}{1 - \alpha_A} \left( \hat{e}_t \hat{p}_t^{*} + (1 - \gamma_1^Q - \gamma_2^Q) \hat{G}_t, \right.
\]
(153)

where

\[
\gamma_1^Q \equiv \frac{a_D p^C C}{a_D p^C C + Q^{BX} + G}.
\]
(154)
\[ \gamma_2^Q \equiv \frac{Q^{DX}}{a_D p^C C + Q^{DX} + G} \]  \hspace{1cm} (155)

The expression obtained for the GDP is now given by

\[ \hat{Y}_t = \gamma_1^Y \hat{Q}_t + \gamma_2^Y (\hat{e}_t \hat{p}_t^N) - \gamma_3^Y (\hat{w}_t \hat{h}_t), \]  \hspace{1cm} (156)

where we first define

\[ \gamma_1^Y \equiv \frac{Q}{Q + p^N N^D} \]  \hspace{1cm} (157)

\[ \gamma_2^Y \equiv \frac{Y}{Y + p^N N^D + Q^{DX}} \]  \hspace{1cm} (158)

\[ \gamma_3^Y \equiv \frac{p^N N^D}{Y + p^N N^D + Q^{DX}} \]  \hspace{1cm} (159)

which we use to define the coefficients of the equation:

\[ \gamma_1^Y = \frac{\gamma_1^Y}{\gamma_2^Y}, \]  \hspace{1cm} (160)

\[ \gamma_2^Y = \frac{1 - \gamma_1^Y}{\gamma_2^Y} \frac{\alpha_A}{1 - \alpha_A} - \frac{1 - \gamma_3^Y}{\gamma_2^Y} \frac{1}{1 - \alpha_A} \]  \hspace{1cm} (161)

and

\[ \gamma_3^Y = \frac{\gamma_3^Y}{\gamma_2^Y}. \]  \hspace{1cm} (162)

The real marginal cost,

\[ \hat{m}_t = \gamma^{MC}_{t-1} + b^q \hat{\tilde{w}}_t + (1 - b^q) \hat{p}^N_t - \hat{e}_t, \]  \hspace{1cm} (163)

where \[ \gamma^{MC} \equiv \frac{\zeta}{1 + p^C}. \]

The relative price of consumption is given by

\[ \hat{p}_t^C = a_{PC} \hat{p}_t^N, \]  \hspace{1cm} (164)

where \[ a_{PC} \] is defined as follows:

\[ a_{PC} \equiv \frac{(1 - a_D) \left( \frac{\theta_C}{\theta_N - T e} \right)^{1 - \theta_C}}{a_D + (1 - a_D) \left( \frac{\theta_C}{\theta_N - T e} \right)^{1 - \theta_C}}. \]  \hspace{1cm} (165)

The inflation rate of consumption goods
\[ \hat{\pi}_t^C = a_{PC} \hat{\pi}_t^N + (1 - a_{PC}) \hat{\pi}_t. \] (166)

Then we have the two policy rules, the interest rate rule given by

\[ \hat{i}_t = h_0 \hat{i}_{t-1} + h_1 \hat{\pi}_t^C + h_2 \hat{Y}_t + h_3 \hat{e}_t, \] (167)

and foreign exchange market intervention rule,

\[ \hat{r}_{t}^{*CB} = k_0 \hat{r}_{t-1}^{*CB} - k_1 (\hat{e}_t - \hat{e}_{t-1}). \] (168)

### 7.2 Reducing the dynamic system

This system of dynamic equations can be reduced considerably.

First, in (126), (127), (128), (166) and (164) we get closed form expressions for the following variables \( \hat{p}_t^N, \hat{\pi}_t^N, \hat{\delta}_t, \hat{\pi}_t^C \) and \( \hat{p}_t^C \), respectively. Therefore, we use these equations to eliminate these variables in order to solve the system.

By using (124) and (152) we get expressions for \( \hat{h}_t \) and \( \hat{w}_t \):

\[
\hat{h}_t = \frac{1 - b^q}{1 + (1 - b^q) \chi} \hat{e}_t - \frac{1 - b^q}{1 + (1 - b^q) \chi} (\sigma \hat{C}_t + \varepsilon_M \hat{i}_t - \hat{z}_t^C) + \frac{1 - b^q}{1 + (1 - b^q) \chi} (\hat{Q}_t - \hat{\varepsilon}_t), \tag{169}
\]

\[
\hat{w}_t = \frac{(1 - b^q) \chi}{1 + (1 - b^q) \chi} \hat{e}_t + \frac{1}{1 + (1 - b^q) \chi} (\sigma \hat{C}_t + \varepsilon_M \hat{i}_t - \hat{z}_t^C) + \frac{\chi}{1 + (1 - b^q) \chi} (\hat{Q}_t - \hat{\varepsilon}_t), \tag{170}
\]

and for the sum of both variables:

\[
\hat{w}_t + \hat{h}_t = \frac{(1 - b^q)}{1 + (1 - b^q) \chi} \hat{e}_t + \frac{b^q}{1 + (1 - b^q) \chi} (\sigma \hat{C}_t + \varepsilon_M \hat{i}_t - \hat{z}_t^C) + \frac{1 + \chi}{1 + (1 - b^q) \chi} (\hat{Q}_t - \hat{\varepsilon}_t). \tag{171}
\]

By inserting equation (149) into (163) and using (170) and (171) we obtain:

\[
\hat{m}_t = \gamma_{MC} \hat{Y}_t i_{t-1} + \left[ \gamma_{MC} \gamma_2 + 1 \right] \frac{(1 - b^q)(1 + \chi)}{1 + (1 - b^q) \chi} \hat{e}_t + \left[ \gamma_{MC} \gamma_2 + 1 \right] \frac{b^q}{1 + (1 - b^q) \chi} (\sigma \hat{C}_t + \varepsilon_M \hat{i}_t - \hat{z}_t^C) + \frac{\gamma_{MC} \gamma_2 + 1}{1 + (1 - b^q) \chi} (\hat{Q}_t - \hat{\varepsilon}_t). \tag{172}
\]
We now use equation (172) to eliminate the variable \( \hat{m}_t \) throughout the system. This leaves the following domestic inflation Phillips equation (substituting into 125):

\[
\hat{\pi}_t = \frac{\beta}{1+\beta} E_t \hat{\pi}_{t+1} + \frac{\beta}{1+\beta} \hat{\pi}_{t-1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\beta)} \gamma^{MC} \gamma^{L}_t \hat{Y}_{t-1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\beta)} [\gamma^{MC} \gamma^{L}_2 + 1] (1-b^q)(1+\chi) \hat{\pi}_t + \frac{(1-\alpha)(1-\alpha\beta) \gamma^{MC} \gamma^{L}_2 (1+\chi) + b^q \chi}{\alpha(1+\beta) [1+ (1-b^q) \chi]} Q_t - \frac{1-(\alpha-\beta)}{\alpha(1+\beta) [1+ (1-b^q) \chi]} \hat{\pi}_t + \frac{(1-\alpha)(1-\alpha\beta) [\gamma^{MC} \gamma^{L}_2 + 1] (1+\chi) - b^q \chi}{\alpha(1+\beta) [1+ (1-b^q) \chi]} \hat{\pi}_t. \tag{173}
\]

A new balance of payment equation (also by using 171):

\[
\gamma^{BP}_1 \tilde{r}_t^{CB} = \gamma^{BP}_1 \tilde{r}_{t-1}^{CB} + \gamma^{BP}_3 \tilde{r}_t^{B} - \gamma^{BP}_3 \tilde{r}_{t-1}^{B} + \left[ \gamma^{BP}_4 \tilde{r}_t^{BP} - \gamma^{BP}_6 \alpha_{PC} - \gamma^{BP}_7 \frac{1-(1-b^q)(1+\chi)}{1+ (1-b^q) \chi} \right] \tilde{\pi}_t - \frac{1-(\alpha-\beta)}{\gamma^{BP}_7 [1+ (1-b^q) \chi]} \tilde{\pi}_t - \frac{b^q}{1+ (1-b^q) \chi} e_{MT} \hat{t}_t - \frac{b^q}{1+ (1-b^q) \chi} \left[ \gamma^{BP}_8 \tilde{r}_t^{B} \delta - \gamma^{BP}_7 \frac{1-(\alpha-\beta)}{1+ (1-b^q) \chi} \right] \tilde{\pi}_t - \frac{b^q}{1+ (1-b^q) \chi} \tilde{\pi}_t \tag{174}
\]

In addition, using (128) the risk adjusted interest rate parity now reads:

\[
\hat{i}_t = E_t \hat{\pi}_{t+1} - \hat{\pi}_t + E_t \hat{\pi}_{t+1} + E_t \hat{\pi}_{t+1}^{t^{NB}} + \gamma^B \left[ (1-\alpha^{RP}) \alpha^{RP}_2 (\hat{\pi}_t + \hat{\pi}_t^{B}) + \alpha^R_2 \hat{\pi}_t^{B} + \hat{\pi}_t \right] \delta, \tag{175}
\]

or

\[
\hat{i}_t = E_t \hat{\pi}_{t+1} - [1- \gamma^B (1-\alpha^{RP}) \alpha^{RP}_2] \hat{\pi}_t + E_t \hat{\pi}_{t+1} + \gamma^B (1-\alpha^{RP}) \alpha^{RP}_2 \hat{\pi}_t^{B} + \gamma^B \hat{\pi}_t^{B} + \gamma^B \alpha^{RP}_2 \hat{\pi}_t^{B} + \gamma^B [1- \gamma^B (1-\alpha^{RP}) \alpha^{RP}_2] \delta. \tag{176}
\]

We substitute the expression for \( \rho_t^C \) in (164) into the domestic good market clearing equation (153) to get:
\[ \hat{Q}_t = \gamma_1^0 \hat{C}_t + \left(\gamma_1^0 a_{PC} + \gamma_2^0 \frac{1}{1-\alpha_A}\right) \hat{e}_t + \gamma_2^0 \frac{1}{1-\alpha_A} \hat{p}_t + (1 - \gamma_1^0 - \gamma_2^0) \hat{G}_t, \quad (177) \]

and, substituting (171) into (156) we get a new expression for the real GDP:

\[ \hat{Y}_t = \gamma_1^0 \hat{Q}_t + \gamma_2^0 (\hat{e}_t \hat{p}_t + (1 - \gamma_1^0 - \gamma_2^0) \hat{G}_t - \hat{G}_t), \quad (178) \]

Furthermore, by using (126) to substitute into (127) to further substitute into (126) and, to finally combine with (167) we can get the following new expression for the equation of the interest rate rule:

\[ \hat{i}_t = h_0 \hat{i}_{t-1} + h_1 [\hat{\pi}_t + a_{PC} (\hat{e}_t - \hat{e}_{t-1})] + h_2 \hat{Y}_t + h_3 \hat{e}_t, \quad (179) \]

or

\[ \hat{i}_t = h_0 \hat{i}_{t-1} + h_1 [\hat{\pi}_t + h_2 \hat{Y}_t + \hat{h}_3 \hat{e}_t - \hat{h}_4 \hat{e}_{t-1}], \quad (180) \]

where

\[ \hat{h}_3 = h_1 a_{PC} + h_3, \quad (181) \]

\[ \hat{h}_4 = h_1 a_{PC}. \quad (182) \]

We now eliminate \( \hat{Q}_t \) (given in equation (177) by substituting it into (173), (174) and (178), which become:

\[ \hat{\pi}_t = b_1 E_t \hat{\pi}_{t+1} + b_2 E_t \hat{\pi}_{t-1} + (b_q \gamma_1^0 + b_C \sigma) \hat{C}_t + \left\{ b_Q \left[ \gamma_1^0 a_{PC} + \gamma_2^0 \frac{1}{1-\alpha_A}\right] + b_e \right\} \hat{e}_t + \\ b_C \epsilon M \hat{i}_t + b_Q \gamma_2^0 \frac{1}{1-\alpha_A} \hat{p}_t + b_Q (1 - \gamma_1^0 - \gamma_2^0) \hat{G}_t - b_C \epsilon \hat{z}_t + b_i \hat{i}_{t-1} - b_e \hat{e}_{t-1}, \quad (183) \]

where

\[ b_1 \equiv \frac{\beta}{1+\beta}, \quad b_2 \equiv \frac{1}{1+\beta}, \quad (184) \]

\[ b_i \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\beta)} \gamma_{MC}^L \gamma_1^L, \quad (185) \]

\[ b_e \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\beta)} \gamma_{MC}^L \gamma_1^L \frac{1}{1+\gamma_{MC}^L}, \quad (186) \]
The expression for real GDP now reads:

\[ b_Q = \frac{(1 - \alpha)(1 - \alpha \beta) [\gamma^{MC} \gamma^L \gamma^e (1 + \chi) + b^g \chi]}{\alpha (1 + \beta)} \frac{1}{1 + (1 - b^q) \chi}, \]  

(187)

\[ b_e = \frac{(1 - \alpha)(1 - \alpha \beta) [\gamma^{MC} \gamma^L + 1](1 - b^q)(1 + \chi)}{\alpha (1 + \beta)} \frac{1}{1 + (1 - b^q) \chi}, \]  

(188)

\[ b_C = \frac{(1 - \alpha)(1 - \alpha \beta) [\gamma^{MC} \gamma^L + 1] b^q}{\alpha (1 + \beta)} \frac{1}{1 + (1 - b^q) \chi}. \]  

(189)

\[ d_1 \tilde{b}^{*B} = d_2 \tilde{b}^{*B}_{t-1} + d_3 \tilde{r}^{*CB} + d_4 \tilde{r}^{*CB} + d_5 \tilde{r}^t - d_6 \tilde{e}_t + \cdots \]

(190)

where

\[ d_1 = \gamma_3^{BP}, \quad d_2 = \gamma_3^{BP}, \quad d_3 = \gamma_1^{BP}, \quad d_4 = \gamma_2^{BP} \]  

(191)

\[ d_i = \gamma_7^{BP} \frac{b^q}{1 + (1 - b^q) \chi} \varepsilon_M, \]  

(192)

\[ d_e = \gamma_4^{BP} - \gamma_6^{BP} a_{PC} - \gamma_7^{BP} \frac{(1 + \chi)}{1 + (1 - b^q) \chi} \left[ (1 - b^q) + \gamma_1^{Q} a_{PC} + \gamma_2^{Q} \frac{1}{1 - \alpha_A} \right], \]  

(193)

\[ d_{e-} = \gamma_5^{BP} \]  

(194)

\[ d_C = \gamma_6^{BP} + \gamma_7^{BP} b^q \sigma + (1 + \chi) \gamma_1^{Q}, \]  

(195)

The expression for real GDP now reads:

\[ \hat{Y}_t = f_C \hat{C}_t + f_i \hat{t} + f_c \hat{e}_t + f_c \hat{e}_t^c + f_G \hat{G}_t + f_i \hat{e}_t + f_{p*} \hat{p}_t \]  

(196)

where

\[ f_C = \gamma_1^{Q} \gamma_1^{Q} - \gamma_3^{BP} \frac{b^q \sigma + (1 + \chi) \gamma_1^{Q}}{1 + (1 - b^q) \chi}, \]  

(197)

\[ f_i = \gamma_3^{BP} \frac{b^q}{1 + (1 - b^q) \chi} \varepsilon_M, \]  

(198)
\[ f_e = \gamma^Y_2 - \gamma^Y_3 \frac{(1 - b^q)(1 + \chi)}{1 + (1 - b^q)\chi} + \left[ \gamma^Y_1 - \gamma^Y_3 \frac{(1 + \chi)}{1 + (1 - b^q)\chi} \right] \left[ \gamma^Q_1 a_{PC} + \gamma^Q_2 \frac{1}{1 - \alpha_A} \right], \quad (199) \]

\[ f_{zc} = \gamma^Y_3 \frac{b^q}{1 + (1 - b^q)\chi}, \quad (200) \]

\[ f_G = \left( \gamma^Y_1 - \gamma^Y_3 \frac{(1 + \chi)}{1 + (1 - b^q)\chi} \right) \left( 1 - \gamma^Q_1 - \gamma^Q_2 \right), \quad (201) \]

\[ f_e = \gamma^Y_3 \frac{(1 + \chi)}{1 + (1 - b^q)\chi}, \quad (202) \]

\[ f_{p^*} = \gamma^Y_2 + \left( \gamma^Y_1 - \gamma^Y_3 \frac{(1 + \chi)}{1 + (1 - b^q)\chi} \right) \gamma^Q_2 \frac{1}{1 - \alpha_A}. \quad (203) \]

By using (196) we can further reduce (190) and get a new balance of payment equation (BP):

\[ d_1 \hat{b}^B_t = d_2 \hat{b}^B_{t-1} + d_3 \hat{r}_{t-1} + d_4 \hat{r}_{t+CB} - d_5 \hat{\tilde{r}}_t - d_6 \hat{\tilde{e}}_t + d_7 \hat{\tilde{c}}_{t-1} + d_8 \hat{\tilde{y}}_t - d_9 \hat{\tilde{G}}_t - d_{10} \hat{\tilde{e}}_t - d_{11} \hat{\tilde{r}}_t - d_{12} \hat{\tilde{c}}_{t-1} + d_{13} \hat{\tilde{c}}_{t} + d_{14} \hat{\tilde{c}}_{t-1}, \quad (204) \]

\[ d_5 = d_i + d_C(f_i/f_c) \quad (205) \]

\[ d_6 = d_e + d_C(f_e/f_c) \quad (206) \]

\[ d_7 = d_e - \gamma^B_{5P} \quad (207) \]

\[ d_8 = d_C(1/f_c) \quad (208) \]

\[ d_9 = d_C(f_G/f_c) - \gamma^B_{5P} \frac{(1 + \chi)(1 - \gamma^Q_1 - \gamma^Q_2)}{1 + (1 - b^q)\chi} \quad (209) \]

\[ d_{10} = \gamma^B_{5P} \frac{(1 + \chi)}{1 + (1 - b^q)\chi} + d_C(f_e/f_c) \quad (210) \]

\[ d_{11} = \gamma^B_{5P} - \gamma^B_{5P} \frac{(1 + \chi)\gamma^Q_2}{1 + (1 - b^q)\chi} \frac{1}{1 - \alpha_A} + d_C(f_{p^*}/f_c) \quad (211) \]

\[ d_{12} = \gamma^B_{5P} \frac{b^q}{1 + (1 - b^q)\chi} + d_C(f_{zc}/f_c) \quad (212) \]

\[ d_{13} = \gamma^B_{5P} \quad (213) \]

\[ d_{14} = \gamma^B_{8P} \quad (214) \]
In addition, we use equation (196) to get an expression for \( \bar{C}_t \) to then substitute it into equation (123) to get a new expression for real GDP, also called an IS equation which is given by:

\[
\tilde{Y}_t = E_t \tilde{Y}_{t+1} - a_1 \left( \hat{\epsilon}_t - E_t \hat{\pi}_t \right) + a_2 E_t \Delta \hat{\pi}_{t+1} + a_4 \Delta \hat{\pi}_t + a_5 \Delta \hat{\pi}_{t+1} - a_6 \Delta \hat{\pi}_{t+1}^c + a_7 \Delta \hat{\pi}_{t+1}^c
\]

(215)

where

\[
a_1 = f_C(1/\sigma), a_2 = f_C(e_M/\sigma) + f_i, a_3 = f_e, a_4 = f_G, a_5 = f_e, a_6 = f_p^* \text{ and } a_7 = f_C(1/\sigma) - f_c.
\]

We also substitute \( \bar{C}_t \) into (183) to get a new expression for the Phillips equation:

\[
\hat{\pi}_t = b_1 E_t \hat{\pi}_{t+1} + b_2 \hat{\pi}_{t-1} + b_3 \tilde{Y}_t + b_4 \hat{\pi}_t + b_5 \hat{\pi}_{t-1} - b_7 \hat{\pi}_t^* - b_8 \hat{\pi}_t^c - b_9 \tilde{Y}_t - b_{10} \hat{\pi}_t
\]

(216)

where

\[
b_3 = \left( b_Q \gamma_1^Q + b_C \sigma \right) (1/f_c),
\]

(217)

\[
b_4 = b_Q \left[ \gamma_1^Q a_{PC} + \gamma_2^Q \frac{1}{1 - \alpha_A} \right] + b_e - \left( b_Q \gamma_1^Q + b_C \sigma \right) (f_e/f_c),
\]

(218)

\[
b_5 = b_C e_M + b_Q \gamma_1^Q \left( f_i/f_c \right),
\]

(219)

\[
b_6 = b_{i-},
\]

(220)

\[
b_7 = \left( b_Q \gamma_1^Q + b_C \sigma \right) (f_p^*/f_c) - b_Q \frac{1}{1 - \alpha_A},
\]

(221)

\[
b_8 = b_C + b_Q \gamma_1^Q + b_C \sigma \left( f_c/f_c \right),
\]

(222)

\[
b_9 = \left( b_Q \gamma_1^Q + b_C \sigma \right) (f_G/f_c) - b_Q (1 - \gamma_1^Q - \gamma_2^Q),
\]

(223)

\[
b_{10} = b_e + \left( b_Q \gamma_1^Q + b_C \sigma \right) (f_e/f_c).
\]

(224)

Finally, the risk adjusted interest rate parity equation now reads:

\[
c_1 \hat{\epsilon}_t = c_2 E_t \hat{\epsilon}_{t+1} + c_2 E_t \hat{\pi}_{t+1} - \hat{\pi}_t + c_3 \hat{b}_t^* + c_4 \hat{\phi}_t^B + c_5 \hat{\phi}_t^{\ast B} - c_2 E_t \hat{\pi}_{t+1}^N
\]

(225)

where

\[
c_1 = \gamma_1^R \left[ 1 - \gamma_1^B (1 - \sigma^{RP}) \sigma_2^{RP} \right]
\]

(226)

\[
c_2 = \gamma_1^R
\]

(227)
7.3 The system in matrix form

In order to write the linearized system in matrix format, we first proceed to define three vectors of variables:

\[ X_t \equiv \begin{bmatrix} \hat{i}_t & \hat{r}^{CB*}_t & \hat{b}^{sB*}_t \end{bmatrix}' \]  

(231)

\[ Y_t \equiv \begin{bmatrix} \hat{Y}_t & \hat{\pi}_t & \hat{e}_t \end{bmatrix}' \]  

(232)

and

\[ Z_t \equiv \begin{bmatrix} \hat{G}_t & \hat{z}^{C}_t & \hat{p}^{s*}_t & \hat{\pi}^{sN}_t & \hat{\epsilon}_t & \hat{i}^{s*}_{t-1} & \hat{\phi}^{sB*}_{t-1} & \hat{z}^{e'}_t \end{bmatrix}' \]  

(233)

Vectors \( X_t \) and \( Y_t \) contain endogenous variables, being \( X_t \) the one with variables without expectational terms and \( Y_t \) containing variables whose equations are associated with expectation operators. Also note that we have added an additional shock \( \hat{z}^{e'}_t \) to study the effects of monetary policy shocks in terms of foreign currency purchases.

The system of six equations can be written in structural matrix form as follows:

\[ B_{11}X_t + B_{12}Y_t = C_{11}X_{t-1} + C_{12}Y_{t-1} + J_1^0 Z_t + J_1^- Z_{t-1} \]  

(234)

\[ B_{21}X_t + B_{22}Y_t = A_{21}E_t X_{t+1} + A_{22}E_t Y_{t+1} + C_{21}X_{t-1} + C_{22}Y_{t-1} + J_2^0 Z_t + J_2^+ E_t Z_{t+1} \]  

(235)

Given that \( E_t Z_{t+1} = M Z_t \), we can write the equations as follows:
\[
\begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
\begin{bmatrix}
X_t \\
Y_t
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
E_t X_{t+1} \\
E_t Y_{t+1}
\end{bmatrix} + \begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix} \begin{bmatrix}
X_{t-1} \\
Y_{t-1}
\end{bmatrix} + \begin{bmatrix}
J_1^0 & J_1^- \\
J_2^0 + J_2^+ M & 0
\end{bmatrix} \begin{bmatrix}
Z_t \\
Z_{t-1}
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix}
\] (236)

We now proceed to define new variables required to write the system in state space form.

Let the matrix (selector matrix) \( S_Y \), be a \( 2 \times 3 \) matrix that select the elements of \( Y_t \) that are lagged and so we define the two dimensional vector
\[
\overline{Y}_t = S_Y Y_{t-1}.
\]

In addition, define the matrices \( \overline{C}_{j2} = C_{j2}S_Y' \) for \( j = 1, 2 \) which have the same elements of \( C_{j2} \) but leaving out all zero columns. Note that, \( \overline{C}_{j2}\overline{Y}_t = C_{j2}Y_t \) for \( j = 1, 2 \).

Therefore, we can now express the system in state space form as follows:

\[
\begin{bmatrix}
I_{3 \times 3} & 0 & 0 & 0 \\
0 & I_{2 \times 2} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\overline{X}_{t+1} \\
\overline{Y}_{t+1}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & I_{3 \times 3} & 0 \\
0 & 0 & 0 & S_Y
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
C_{11} & -C_{12} & B_{11} & B_{12} \\
-C_{21} & -C_{22} & B_{21} & B_{22}
\end{bmatrix}
\begin{bmatrix}
\overline{X}_t \\
\overline{Y}_t
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
-(J_2^0 + J_2^+ M)
\end{bmatrix}
\begin{bmatrix}
Z_t \\
Z_{t-1}
\end{bmatrix}
\] (237)

where

\[
\overline{X}_t \equiv \begin{bmatrix}
\hat{i}_{t-1} \\
\hat{r}_{t-1}^{*CB} \\
\hat{b}_{t-1}^{*B}
\end{bmatrix}',
\]

\[
\overline{Y}_t \equiv \begin{bmatrix}
\hat{\pi}_{t-1} \\
\hat{e}_{t-1}
\end{bmatrix}'
\]

and

\[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

In addition, the first autoregressive equation for the forcing stochastic processes can be written as:

65
\[
\begin{bmatrix}
Z_t \\
Z_{t-1}
\end{bmatrix} = 
\begin{bmatrix}
M & 0 \\
0 & M
\end{bmatrix}
\begin{bmatrix}
Z_{t-1} \\
Z_{t-2}
\end{bmatrix} + 
\begin{bmatrix}
\rho_t \\
\rho_{t-1}
\end{bmatrix}
\]  

(238)

where \(\rho_t\) \(i.i.d\) \(N(0, \Sigma)\) and \(M\) is a square matrix congruent with \(Z\). We further assume that the exogenous shocks are independent to each other and follow \(AR(1)\) processes. In particular, we assume that

\[
\ln \varepsilon_t = \rho_{\varepsilon} \ln \varepsilon_{t-1} + \nu_{t}^{\varepsilon}
\]

(239)

\[
\ln G_t = \rho_G \ln G_{t-1} + \nu_{t}^{G}
\]

(240)

\[
\ln z_{C_t}^C = \rho_{zC} \ln z_{C_{t-1}}^C + \nu_{t}^{zC}
\]

(241)

\[
\ln z_{r_t}^C = \rho_{zr} \ln z_{r_{t-1}}^C + \nu_{t}^{zr}
\]

(242)

\[
\ln \phi_{B_t} = \rho_{\phiB} \ln \phi_{B_{t-1}} + \nu_{t}^{\phiB}
\]

(243)

\[
\ln p_{t}^* = \rho_{p^*} \ln p_{t-1}^* + \nu_{t}^{p^*}
\]

(244)

\[
\ln p_{t}^* = \rho_{p^*} \ln p_{t-1}^* + \nu_{t}^{p^*}
\]

(245)

\[
\ln \pi_{N_t}^* = \rho_{\piN} \ln \pi_{N_{t-1}}^* + \nu_{t}^{\piN}
\]

(246)

\[
\ln i_t^* = \rho_{i^*} \ln i_{t-1}^* + \nu_{t}^{i^*}
\]

(247)

with all the persistence parameters \((\rho^*)\) are positive and less than one, and the \(\nu_t^i\); are \(i.i.d\). shocks.

We can now write the system in its state space representation which reads:

\[
A \varepsilon_{t+1} = B \varepsilon_t + J \tilde{Z}_t,
\]

(248)

\[
\tilde{Z}_t = M_t \tilde{Z}_{t-1} + \tilde{\rho}_t
\]

(249)

which can be used to apply the generalized Schur method proposed in Klein (2000) to solve this rational expectation model. Briefly, As long as there exists some complex \(\sigma\) such that \(\text{det}(A \sigma - B) \neq 0\), there exist unitary matrices of complex numbers, \(Q\) and \(Z\), such that \(QAZ \equiv S\) and \(QBZ \equiv T\) are upper triangular and such that for all \(i\) the diagonal elements \(S_{ii}\) and \(T_{ii}\) are not both zero. Also, the set of generalized eigenvalues is the set of ratios \(T_{ii}/S_{ii}\) where, when \(S_{ii} = 0\) we call the corresponding generalized eigenvalue “infinite”. Furthermore, the pairs \((S_{ii}; T_{ii})\) can be arranged in any order. Hence the eigenvalues can be arranged so that the ones within the unit disk come first.
We form a partition of the vector $x_t$ with dimension $nx = 11$. The state variables are included in a vector $k_t$ (the ones appear lagged, i.e. with dimension $nk = 5$) and the remaining endogenous variables in $d_t$:

$$x_t = \begin{bmatrix} \bar{X}_t \\ \bar{Y}_t \\ X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} k_t \\ d_t \end{bmatrix},$$

where

$$k_t = \begin{bmatrix} \bar{X}_t \\ \bar{Y}_t \end{bmatrix}$$

and

$$d_t = \begin{bmatrix} X_t \\ Y_t \end{bmatrix}$$

According to Klein (2000), if the upper left block of $Z$, called $Z_{11}$ which contains $nk$ rows and columns is non-singular, and the number of eigenvalues within the unit circle (i.e. the number of $i \in (1, \ldots, nx)$ such that $|T_{ii}| < |S_{ii}|$) is equal to the dimension of $k_t$, then for any $k_0$ there exist a unique solution that can be expressed as follows:

$$k_{t+1} = Gk_t + H\tilde{Z}_t + \xi_{t+1}$$
$$d_t = Kk_t + L\tilde{Z}_t,$$

where $\xi_{t+1}$ is a martingale difference process and

$$G = Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}$$
$$H = (GZ_{12} - Z_{11}S_{11}^{-1}T_{12})R + (Z_{11}S_{11}^{-1}S_{12} - Z_{12})RM + Z_{11}S_{11}^{-1}Q_1J$$
$$K = Z_{21}Z_{11}^{-1}$$
$$L = (KZ_{12} - Z_{22})R$$
$$\text{vec}(R) = \left[I - \tilde{M} \otimes (T_{22}^{-1}S_{22})\right]^{-1} \text{vec}(T_{22}^{-1}Q_2J),$$

where $Z_{ij}$, $S_{ij}$ and $T_{ij}$ are the blocks of the matrices $Z$, $S$ and $T$, respectively that corresponds to their partition and $Q_1$ and $Q_2$ are the corresponding upper and lower blocks of the matrix $Q$.

We can integrate the exogenous autoregressive processes with the model solution to form the following expression which is the one used for the impulse response analysis:
\[
\begin{bmatrix}
k_{t+1} \\
\tilde{Z}_{t+1}
\end{bmatrix} = \begin{bmatrix} G & H \\ 0 & \tilde{M} \end{bmatrix} \begin{bmatrix} k_t \\ \tilde{Z}_t \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \tilde{\rho}_{t+1}
\]

\[
[d_t] = [K \quad L] \begin{bmatrix} k_t \\ \tilde{Z}_t \end{bmatrix}
\]