Phillips Curve in Brazil: an unobserved components approach

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Resumo
Este artigo apresenta estimações da Curva de Phillips para o Brasil, utilizando a abordagem de séries de tempo com componentes não-observados. A decomposição em tendência, sazonalidade e ciclo permite interpretações econômicas diretas. De forma diferente de Harvey (2008), incluímos as expectativas de inflação em uma especificação similar a uma Curva de Phillips novo-Keynesiana híbrida empregando uma série ainda não-explorada no Brasil, que é o IBC-Br, como proxy para o produto. Em seguida um modelo multivariado de inflação e produto com componentes não-observados é ajustado, supondo que ambos seguem ciclos similares. Conclui-se que o regime de metas de inflação no Brasil tem conseguido reduzir a variância tanto da sazonalidade como do nível da inflação. Além disso, todas as medidas de atividade econômica empregadas parecem ter respondido cada vez menos à inflação nos anos mais recentes, embora em alguns casos o intervalo de confiança foi considerável. Tal fato é evidência de um achatamento da Curva de Phillips no Brasil, tendência também mostrada por Tombini & Alves (2006), o que significa maiores custos desinflacionários por um lado, mas também menores pressões sobre a inflação derivadas de crescimento do PIB.

Palavras-chave: Curva de Phillips novo-Keynesiana, Filtro de Kalman, componentes não-observados, inflação

Área 3: Macroeconomia, Economia Monetária e Finanças

Abstract
This paper presents estimations of the reduced-form Phillips Curve for recent Brazilian data, using a framework of unobserved components time series models. The decomposition into trend, seasonal and cycle features offers, through the graphical output, straightforward economic interpretations. Differently from Harvey (2008), we allow for inflation expectations in a specification similar to a hybrid new Keynesian Phillips Curve and we also use an unexplored time series, which is the IBC-Br, as a proxy for GDP. Then, a multivariate unobserved components model of inflation and output is fitted, assuming that they follow similar cycles. Our findings support the view that Brazilian inflation targeting has been successful in reducing the variance of both the seasonality and level of the inflation rate. Furthermore, all measures of economic activity employed seem to have responded progressively less to inflation in recent years, although in some cases large confidence intervals were found. This provides some evidence of a flattening of the Phillips Curve in Brazil, a trend also shown by Tombini & Alves (2006), which translates into higher costs of disinflation on the one hand, but also lower inflationary pressures derived from output growth, on the other hand.

Keywords: New Keynesian Phillips Curve, Kalman filter, unobserved components, inflation

JEL Classification: C32, E31

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1. Introduction

The Phillips curve (PC), ever since the first approaches developed by A. W. Phillips (1958) and Samuelson & Solow (1960), has been a constant subject of debate in macroeconomics. Its implicit formulation encompasses an important trade-off between inflation rate and unemployment rate or alternatively between inflation rate and output gap. Numerous countries use this aggregate supply relation when formulating and implementing monetary policy, often jointly with an aggregate demand equation (IS) and an interest rate rule. The PC is also commonly utilized in inflation forecasting models, as reviewed by Stock & Watson (2008).

In the past few years the new Keynesian PC has become popular in its various forms. The initial form, mainly attributed to Calvo (1983), consisted of a connection between inflation and real marginal cost plus an inflation expectation component. The driving force was the observed sticky price adjustment by some firms. Over time, some changes occurred, often to make up for flaws in microeconomic (approach to price adjustments), empirical (i.e. persistence of observed inflation, not contemplated in the original equation) or macroeconomic aspects (under some circumstances, the coefficient of real marginal cost may be negative, which is economically counterintuitive, as Rudd & Whelan (2007) mention). Even the widely adopted hybrid new Keynesian PC, which includes lagged inflation, has not been successful in explaining inflation dynamics in a satisfactory manner. According to Rudd & Whelan (2007), this occurs whether output gap or labor income is used as a proxy for real marginal cost. Mavroeidis (2005) assesses a few methodological problems with estimating forward-looking rational expectations models by GMM. By focusing mainly on the model proposed by Gali & Gertler (1999), the author concludes that limited information methods, such as GMM, usually do not account for identifiability conditions. In other words, the controversies over the econometric approach to the new Keynesian PC, combined with its theoretical relevance, open up opportunities for different approaches.

Therefore, it is possible to say that, despite some recent renewed interest in the PC, supported by the new Keynesian literature, the dynamics of the relation between inflation and economic activity expressed by it is deprived of strong empirical or theoretical foundations. The efficiency of its estimations for the recent past of the Brazilian economy, as pointed out by Sachsida et al (2009), is equally fuzzy.

Given the clear-cut empirical difficulties of such an important relation for economic theory as the PC, in the present study we verify the dynamics of inflation in an estimation with unobserved components for the Brazilian economy, following to some extent the approach proposed by Harvey (2008). A simple relationship is established between monthly inflation and output data, in which inflation is explained by a set of unobserved components (UC), in addition to the usual output gap and expected inflation terms. The first term is identified from detrended output, also by the UC method. Differently from Harvey (2008), the expectations term is introduced in the PC, bringing the model closer to a standard hybrid new Keynesian PC. This way, notions about price rigidity and

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3 Some authors (Gali & Gertler, 1999 and Schwartzman, 2006 and Sachsida et al., 2009 in the Brazilian case) suggest that the output gap has not been a significant measure of inflationary pressures in GMM estimations. On the other hand, measures such as labor income, or unit labor cost are also criticized for they have a countercyclical pattern in the analysis of U.S. data (Rudd & Whelan, 2007). As in the present study we use a different method from that which is commonly adopted, we would rather test the output gap, which is an important measure of policy for most central banks.
inflationary inertia are taken into account, but at the same time, we depart from standard econometric approaches to the PC. The stochastic trend component, modeled as a random walk, is regarded as core inflation, successfully substituting the lagged term of the hybrid new Keynesian PC. Thereafter, a multivariate estimation is used, in which output gap is implicitly present in the output equation, no longer being inserted exogenously, which is an advantage as it precludes the previous estimation of an additional unobserved component. Put differently, the PC parameters are found without having to first estimate the output gap.

The aim of the present model is therefore to parsimoniously reproduce the stylized facts of the relation between inflation and output gap, providing more recent estimates of this important interaction and its dynamics for Brazil. A further contribution concerns the use of output gap obtained from detrending Central Bank’s index of economic activity (IBC-Br), still unexplored in academic works. Notwithstanding the relatively small sample size, interesting conclusions can be drawn from this series, which leads to some reflection about the contemporaneous evolution of economic activity in Brazil.\footnote{This index was adopted by the Central Bank of Brazil in 2009, in order to follow up economic activity in a more tempestive fashion, due to its low occurrence of lags and to its monthly periodicity. According to the Central Bank’s Inflation Report of March 2010, IBC-Br is considerably attached to the GDP series. Besides new estimations, back-calculations starting in January 2003 were made and used in the present study.}

By also considering time variation of the output gap parameter, which is new in comparison to Harvey (2008), we also test its linearity for Brazil. Some studies on developed countries, as the one conducted by Kuttner & Robinson (2008), advocate the recent flattening of the PC, in the sense that the output gap parameter has become gradually smaller. This behavior has important macroeconomic implications, as will be discussed later. Moreover, an analysis of the forecasting power is carried out by comparing our models with a simple forecasting model in order to test the assumption that Phillips curves may provide good inflation forecasts (Stock & Watson, 2008).

Much of the literature that focus on the estimation the new Keynesian PC considers the inflation trend to be stationary, as reviewed by Rudd & Whelan (2007) and Nason & Smith (2008b). On the other hand, recent works have sought to model Phillips curves with a stochastic inflation trend, as done in the present study. Lee & Nelson (2007) propose a bivariate specification between inflation and unemployment, in which inflation trend varies over time. Goodfriend & King (2009) explain the stochastic behavior of inflation trend based on assumptions about the Central Bank’s policy.

Dealing more specifically with the PC with unobserved components, Vogel (2008) uses a modeling strategy that resembles the one utilized in the present study, but instead she regarded the unemployment gap as a variable of inflationary pressures. Interestingly, her work combines the idea of Gordon’s (1997) “triangle” model of inflation, in which the NAIRU varies over time, with the new Keynesian model that focuses on short-term inflation dynamics. Harvey (2008) proposes decomposing inflation into transitory and permanent components, following the methodology of structural time series models, described in more detail in Harvey (1989).

With respect to the Brazilian literature on this issue, Sachsida et al (2009) provide a good survey and propose a regime-switching model to account for time-variation in the PC parameters. Schwartzman (2006) estimates the PC using industrial capacity utilization data to address the fact that the output gap is not observable. Fasolo & Portugal (2004) adapt a new Keynesian PC for
Brazil based on the NAIRU, giving a sharper focus on expectations formation. Some works, such as Arruda et al (2008) and Correa & Minella (2005), used PC versions to assess their inflation forecasting power. To our knowledge, there are no Brazilian studies that investigate inflation dynamics with a primary focus on the decomposition of its factors using permanent and transitory unobserved components.

The paper is organized into five sections. Section 2 deals with conceptual issues and with the econometric estimation of the PC with exogenous marginal cost measures. Section 3 presents the multivariate estimation and its respective results. Section 4 describes some extensions to the basic model and Section 5 concludes.

2. Phillips curve basic model with unobserved components

The PC proposed in the model partially follows Harvey (2008). The author used a structural time series approach in which the gap was regarded as explanatory variable in a decomposition of the inflation rate.

The specification with unobserved components has some advantages over ARMA models. First, unlike the ARMA specification, the components provide a straightforward economic interpretation. More importantly, in ARMA specifications, the model’s dynamics relies exclusively upon the dependent variable, whereas in UC models, the dynamics is constantly inferred by observations.5

Before moving on to the specification used, some brief comments should be made about Harvey’s (2008) model and about the adaptations made.

A basic structural6 time series model can be easily represented by:

$$\pi_t = \mu_t + \psi_t + \gamma_t + \varepsilon_t$$

where the observed series $\pi_t$ is decomposed into trend ($\mu_t$), cycle ($\psi_t$), and seasonality ($\gamma_t$), components, and into an irregular white noise component ($\varepsilon_t$). In addition to permanent and transitory components, it is possible to add explanatory variables, as well as structural breaks and breaks in the slope and outliers, as in a usual regression.

When one includes an output gap term $h_t$ in equation (1), as a measure of inflationary pressure, there is a Phillips curve similar equation, using unobserved components:

$$\pi_t = \mu_t + \psi_t + \gamma_t + \phi h_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$$

For Harvey (2008), under some hypotheses, an inflation model with this configuration may simultaneously capture the backward- and forward-looking ideas of the hybrid new Keynesian PC.7

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5 This issue was dealt with by Wongwachara & Minphimai (2009).
6 Models with unobserved components are also known in the literature as structural time series models. See Harvey (1989).
7 Gali & Gertler (1999) and Christiano, Eichenbaum & Evans (2005) are theoretical references on the treatment of inflation through the hybrid new Keynesian PC. The differences basically lie in the way prices are adjusted and in their nominal rigidity.
This configuration is based on the following terms explaining current inflation: lagged inflation, output gap and a future inflation expectation component, i.e.:  

\[ \pi_t = \delta_0 \pi_{t-1} + \phi h_t + \delta_f E_t(\pi_{t+1}) + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2) \]  

(3)

In other words, Harvey (2008) focuses on estimating (2), arguing that this formulation contemplates the notion of a hybrid PC as in (3).

At least with respect to the lagged term, it is reasonable to affirm that it can be successfully replaced with the specification proposed here. It suffices to observe that a simple model that combines inflation and output gap \( h_t \):

\[ \pi_t = \mu_t + \phi h_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2) \]  

(4)

\[ \pi_t = E_{t-1}(\mu_t) + \phi h_t + \nu_t \quad \nu_t \sim N(0, \sigma^2) \]  

(5)

where \( \nu_t = \pi_t - E_{t-1}(\pi_t) \) is an innovation and \( E_{t-1}(\mu_t) \) is a weighted average of past observations, corrected for the output gap’s effect. If we include cycle and/or seasonality components in (5), we have the term \( E_{t-1}(\mu_t) \) capturing not only the past trend, but also the information on lagged inflation rates, appropriately weighted. Economically, this formulation seems to be more realistic than the PC with a lagged inflation term. In addition, as pointed out by Harvey (2008), admitting that \( h_t \) is stationary in (4), the long-term inflation forecast is the current value of \( \mu_t \), i.e., the unobserved term of the structural model becomes a measure of core inflation or underlying rate of inflation.

In regard to the expectations term, our view is different from that adopted by Harvey (2008), which turned a hybrid new Keynesian PC, like equation (3), into an equation that relates inflation to core inflation expectation, to future output gaps expectation and to the current output gap, i.e.,

\[ \pi_t = E_{t-1}(\mu_t) + \gamma \phi^* \sum_{j=0}^{\infty} \gamma^j E_t(h_{t+1+j}) + \phi h_t + \nu_t \quad \nu_t \sim N(0, \sigma^2) \]  

(6)

In addition to the need to appeal to several simplifying assumptions, this does not fully solve the problem, that is, it does not allow, in general terms, modeling the past and future effects of hybrid NKPC as in the equation with unobserved components, or in the present model, equation (4). The author acknowledges the difficulty in doing so and places little importance on the future term, citing Rudd & Whelan (2007) and Nason & Smith (2008a).

Unlike Harvey (2008),\(^9\) we included the future inflation expectations term in the analysis, as we consider it to be a crucial element in the modeling of inflation dynamics when price rigidity is assumed. Furthermore, it is possible to check whether the future term does play a major role for the

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8 Nason & Smith (2008b) argue that the hybrid new Keynesian PC is consistent with a variety of price and information adjustment schemes. Therefore, the focus on reduced-form coefficients, \( \delta_0, \phi \in \delta_f \), instead of on structural parameters, simplifies the analysis without interfering in the importance of the result.

9 Vogel (2008) also argues that inflation expectations should not be neglected, citing the difficulty in the identification of \( \mu_t \) in Harvey (2008) regarding past or future effects.
Brazilian data in our model, as highlighted by Sachsida et al (2009). There is also a debate on the appropriate measure of inflation expectations. Nason & Smith (2008b) draw attention to the weak identification of GMM-based estimates, which strengthen the use of survey data. On the other hand, some authors highlight the drawback of survey-based forecasting bias, as a sign of agents’ lack of rationality. However, Araujo & Gaglianone (2010) state that the Focus Bulletin series do not suffer forecasting bias in the case of expectations over a shorter time horizon (one and three months ahead). Among the studies that used inflation expectations surveys, Basistha & Nelson (2007), for instance, adopt an inverse perspective, in which they estimate the output gap using a forward-looking PC.

As usual, in the empirical literature, one should also consider some real activity variable that represents the inflationary pressure (or the real marginal cost, as in the original new Keynesian PC). The most frequent examples include labor income share, deviation of the natural rate of unemployment or output gap. In the present study, the major focus is on the output gap, measured by two indicators, the gross domestic product and the IBC-Br series, developed by the Central Bank of Brazil (CBB). Even though some authors such as Schwartzman (2006) and Sachsida et al (2009) support series with larger economic information to the detriment of econometrically detrended gap series, we use the output gap for the following reasons: it is still an important index used by the Central Bank for monetary policy formulation as an indicative sign of demand pressures on prices. Secondly, as this is a new method for the analysis of Brazilian data, it should be tested in comparison to this index, which is widely used in economic theory and practice. Last but not least, it is expected that with the gradual and larger availability of data after the introduction of the inflation targeting system, the output gap may become more representative of inflationary pressures in Brazil. Nonetheless, for the sake of comparison, we reproduced the same estimations with the monthly industrial capacity utilization (ICU) series.

A large strand of the literature is devoted to the estimation of the output gap series, which is not directly observed in the economy. Since the primary goal is not to explore these techniques, we opted for the decomposition of the logarithm of output into unobserved trend and cycle components, also used in Harvey (2008), but closer to what Gerlach & Peng (2006) did. Simultaneously, we extracted the seasonality of the series with the component $y_t$:

\[
\log y_t = \mu_t + \gamma_t + \psi_t + \varepsilon_t
\]

\[
\mu_t = \mu_{t-1} + \beta_{t-1}
\]

\[
\beta_t = \beta_{t-1} + \zeta_t
\]

and the stochastic cycle $\psi_t$, which is equivalent to the output gap $h_t$ of the PC in the modeling and takes on the following form:

\[
\begin{bmatrix}
\psi_t \\
\psi_t^*
\end{bmatrix} = \rho \begin{bmatrix}
\cos \lambda_c & \sin \lambda_c \\
-\sin \lambda_c & \cos \lambda_c
\end{bmatrix} \begin{bmatrix}
\psi_{t-1} \\
\psi_{t-1}^*
\end{bmatrix} + \kappa_t
\]

According to this model, studies that consider the PC to be nonlinear underestimate the role of the future term in the Brazilian inflation dynamics.

The main paths are: production function approach, which has the advantage of imposing some economic structure, with information on capital accumulation and on total factor productivity; and the econometric approach, in which the trend of the real GDP series is identified as the potential output, a good and useful approximation when good macroeconomic data on capital and labor are not available.
where $\rho_\psi$ is a damping factor, $\lambda_c$ in the frequency in radians ($0 \leq \lambda_c \leq \pi$). Error terms $\epsilon_t$ and $\zeta_t$ are normally independently distributed with variances $\sigma_\epsilon^2$ and $\sigma_\zeta^2$. $\kappa_t$ and $\kappa_t'$ are mutually uncorrelated disturbances with 0 mean and common variances $\sigma_\kappa^2 = \sigma_\kappa'^2$. The dynamics of the seasonal stochastic component $\gamma_t$ is identical with the one described in equations (14) and (15). Note that the expression above indicates a smooth trend which, together with a cyclical component, represents an attractive decomposition for output data, according to Koopman et al (2007). The trend described in (8) and (9) can be also referred to as integrated random walk. The Hodrick-Prescott (HP) filter is another traditional tool for trend extraction. However, even if the resulting output gap is similar to the one obtained here, the HP filter tends to be less efficient at the end of the series, as described by Mise et al (2005).

The estimations carried out begin with a simpler PC (model I), adapted from Harvey (2008), with the inclusion of interventions in order to capture irregularities in the data:

$$
\pi_t = \mu_t + \gamma_t + \phi h_t + \sum_{k=1}^l d_k \theta_{k,t} + \epsilon_t \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)
$$

where $\mu_t$ and $\gamma_t$ follow the same dynamics of equations (13) through (15).

With the inclusion of the expectations term, one obtains the proposed PC model, which is classified as model II, III or IV in the next section, depending on the variable used as measure of marginal cost:

$$
\pi_t = \mu_t + \gamma_t + \phi h_t + \delta_f \pi_{t+1} + \sum_{k=1}^l d_k \theta_{k,t} + \epsilon_t \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)
$$

$$
\mu_t = \mu_{t-1} + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2)
$$

$$
\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{j,t} \quad t = 1, \ldots, T
$$

where each $\gamma_{j,t}$ is generated by:

$$
\begin{bmatrix}
\gamma_{j,t} \\
\gamma_{j,t-1}
\end{bmatrix} = \begin{bmatrix}
\cos \lambda_j & \sin \lambda_j \\
-\sin \lambda_j & \cos \lambda_j
\end{bmatrix} \begin{bmatrix}
\gamma_{j,t-1} \\
\gamma_{j,t-1}
\end{bmatrix} + \begin{bmatrix}
\omega_{j,t} \\
\omega_{j,t}'
\end{bmatrix} \quad j = 1, \ldots, [s/2]
$$

In the above expression for trigonometric seasonality, $\lambda_j = 2\pi j/s$ is the seasonal frequency in radians, and $\omega_t$, $\omega_t'$ are normally independent distributed seasonal disturbances with zero mean and common variance $\sigma_\omega^2$. To choose the intervention dummy variables $\theta_{k,t}$ we analyzed the auxiliary residuals, which are smooth estimates of the disturbances of irregular, level and slope components.

Equation (12) is also called measurement or observation equation with variables that explain the observed inflation. Equations (13) through (15) form the state equations that characterize the dynamics of unobserved variables. Note that the inflation trend is dealt with by using a local level

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12 Portugal (1993) performs a similar estimation for annual data from 1920 to 1988, but he first deals with the fixed output growth rate and then with the stochastic one. In the present model, the difference between both is the term $\zeta_t$.

13 The inclusion of the cyclical component $\psi_t$ was also tested, but it was found to incorrectly capture some typical outlier episodes such as peaks or troughs found in the cycles. A larger amount of years would likely minimize this problem. Thus, the component was not considered at this estimation stage.
approach, compatible with nonstationarity, which is common in the literature. As to seasonality, the component $\gamma_\tau$ can be seen as the sum of time-varying trigonometric cycles.

For implementation of the Kalman filter algorithm, basically it is necessary that the model’s equations be in state-space form, i.e.:

$$
\pi_t = (1 \ 1 \ 0) \alpha_t + \phi h_t + \delta_f E_t(\pi_{t+1}) + d_k \theta_{k,t} + (\sigma_x \ 0 \ 0 \ 0) u_t
$$

$$
\alpha_t = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \lambda_j & \sin \lambda_j \\
0 & -\sin \lambda_j & \cos \lambda_j
\end{pmatrix} \alpha_{t-1} + \begin{pmatrix}
0 & \sigma_\eta & 0 & 0 \\
0 & 0 & \sigma_\omega & 0 \\
0 & 0 & 0 & \sigma_\omega
\end{pmatrix} u_t
$$

where $\alpha_t = (\mu_t \ \gamma_{j,t} \ \gamma_{j,t}')'$ and $u_t = (\epsilon_t \ \eta_t \ \omega_t \ \omega_\tau)'$.

Summarizing the basic ideas about the model, it resembles a reduced-form new Keynesian PC, with future inflation term and output gap as explanatory variables. Nevertheless, it also captures, to some extent, past inflation behavior by way of the decomposed trend and seasonality terms, in an attempt to mitigate an empirical deficiency that is commonly referred in the literature.\(^{14}\)

### 2.1 Data and the econometric approach

In order to estimate equations (11) through (15) and the subsequent steps of this work, we used the following series, which are summarized in Table 1.

<table>
<thead>
<tr>
<th>Monthly data series – April/2000 through March/2010</th>
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<tbody>
<tr>
<td><strong>Variable</strong></td>
</tr>
<tr>
<td>Inflation</td>
</tr>
<tr>
<td>Output</td>
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<tr>
<td></td>
</tr>
<tr>
<td>Inflation expectations</td>
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<tr>
<td>Marginal cost in PC</td>
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<tr>
<td>Industrial capacity utilization</td>
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</table>

Notes: The IBC-Br series is only available after January 2003. IPCA: Broad consumer price index; IBGE: Brazilian Institute of Geography and Statistics; PIB-BC: Monthly output series published by the BCB; BCB: Brazilian Central Bank; IBC-Br: BCB’s index of economic activity; CNI: National Confederation of Industry;

**Table 1: Data series**

In practice, monthly GDP series with current prices, from April 2000 to March 2010 (source: [http://www4.bcb.gov.br/?SERIESTEMP].\(^{15}\)) was decomposed, following equations (7) through (10), using the Kalman filter algorithm in the OxMetrics 5 package (STAMP module). The trend component obtained from the estimation is a good approximation for the potential output in

\(^{14}\) Fuhrer & Moore (1995) were the first to argue that the standard new Keynesian price adjustment models could not explain the persistence in the empirical process of inflation.

\(^{15}\) The BCB interpolates IBGE’s official quarterly series to obtain a monthly series.
the period. The difference between the observed series and its deseasonalized trend is the output gap. In this case, disregarding the error term, as its variance was very close to zero, the cyclical component corresponds to the output gap. Similar reasoning was used to extract the output gap from the IBC-Br series. Figure 1 shows the comparison of the series obtained from the PIB-BC gap, from the IBC-Br gap and from the relative deviations from the ICU, all of which are expressed as percentage. Note that the gap, as specified, varies between positive and negative percentage values. The ICU series was collected from the IPEADATA database, and the percentage difference between the real series and the average for the period (calculated as 80.87%) was used to construct the deviations.

Inflation expectations were obtained from the Central Bank’s FOCUS survey, using the available 10-year series. In the present study, we used the median of daily expectations within each month relative to the forward month.

![Figure 1 – Comparison of economic activity series](image)

With the output gap and industrial capacity utilization series for the period at hand, we carried out the following PC estimations.

First, we tested a model that is similar to the one used in Harvey (2008), which consists of equation (11) and is identified in Table 2 as model I. Afterwards, to highlight the fact that it is important to introduce inflation expectations in the modeling of the PC with unobserved

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16 This occurs because the series trend, by definition, crosses the real series in different moments. Theoretically, the positive gap values correspond to moments of heated economic activity, which result in inflationary pressures.
components, we use equation (12). In these first two alternatives, the measure of marginal cost used is the output gap calculated from the monthly GDP provided by the BCB. The third model concerns a PC that is identical to equation (12), but with ICU data instead of output gap. Finally, the IV model consists of the same equation (12) with the difference that the output gap series was calculated using IBC-BR. Note that, due to some unusual inflation movements, especially between 2002 and 2003, the inclusion of interventions inevitably leads to a better fit.

The evaluation of models follows some fitting and residuals diagnostic statistics. With respect to fitting, the chief indicators contemplated in the estimation of the output gap and of the PC were the following: algorithm convergence, forecast error variance decomposition, and log-likelihood. According to Koopman et al. (2007), a good convergence is key to showing that the model was properly formulated and that, in general, it does not have fitting problems. Prediction error variance (PEV) is the basic measure of goodness-of-fit which, in steady state, corresponds to the variance of the one-step-ahead forecast errors. Other diagnostic statistics analyzed include Box-Ljung’s Q statistics, for the assessment of residuals autocorrelation, and normality (N) and heteroskedasticity (H) results.

2.2 Results

In the output gap estimation, a “very strong” convergence and a relatively small prediction error variance were obtained. The recent global financial crisis and the resulting sharp decrease of both output gap measures and of the deviations in the ICU in the last quarter of 2008 is noteworthy.

Table 2 shows the different models described in Section 2.1, following an increasing order from the worst to the best fit. In all cases, convergence was again “very strong,” satisfying the principal modeling criterion proposed by Koopman et al. (2007). The prediction error variance decreased from I to IV, as expected, indicating superior fit of the models (II through IV) that include inflation expectations. The log-likelihood indicators underscore this conclusion, as they increased from I to III. In case of model IV, the reduction is more a result of sample size than of the goodness of fit, given that log-likelihood is an absolute and cumulative indicator.

The traditional coefficient of determination undergoes a slight change in case of seasonal data, $R^2$, and measures the relative performance of the specified model in relation to a simple random walk model with drift and fixed seasonality. Again, the result is better for models II and III. According to Box-Ljung’s $Q$ statistics, serial correlation of residuals is absent in all models and significance is lower than 0.1%.

With values lower than one, heteroskedasticity tests indicate that the variance of residuals decreases slightly over time. Unequivocally, this results from the improvement of the inflation targeting regime in Brazil, with an increasingly larger convergence of the inflation rate towards the targets. Even in model IV, with a more recent sample, the pattern is maintained in favor of lower variances for the more recent months. As to normality, the models clearly succeeded on the test, based on Doornik-Hansen’s statistic whose critical value at a 5% significance level is 5.99.
Table 2: Phillips Curve estimation results

The slopes $\phi$ of the different Phillips curves – which are the coefficients for output gap and ICU deviation at the end of the sample – were positive in all cases, but not statistically significant in case II and III. The same comment made by Tombini & Alves (2006), that smaller coefficients than most of those described in the literature are due to the monthly frequency of data, applies here. In regard to the coefficients of inflation expectations, the values showed high statistical significance. Values are slightly larger than one, suggesting alignment of inflation expectations with the observed rates.

In brief, test statistics indicate an improvement in the fit when going from an approach without inflation expectations, as in Harvey (2008), to an approach that includes them. Among output gap measures of the monthly GDP and the ICU deviations, there is no clear superiority of fitting. In both cases, coefficients were positive, as recommended by the economic theory, though not statistically significant. On the other hand, the output gap measure calculated based on the IBC-Br series was positively correlated with the inflation rate, with a high level of significance, although the amount of available data is smaller.
Figure 2 shows the decomposition obtained in model II, whose performance was clearly better than that of the model adapted from Harvey (2008). The first graph compares observed inflation values with the composition of trend and of the regression and intervention effects.

The middle graph shows the seasonal effects. The values correspond to the absolute contribution due to seasonality. For instance, a value of 0.2 indicates that in that month 0.2% of price fluctuation is exclusively due to the seasonal effect of the month. Note also that the variance of this effect decreases in more recent years, with a sharp increase in the effect of February over the last two years. Finally, the lower graph shows the irregular component. Considering that the period between late 2002 and mid-2003 had the three largest discrepant observations regarded as outliers, this graph also depicts some gradual reduction in the variability of disturbances.

3. Multivariate estimation

Complementing the previous analysis, we fitted a bivariate PC, in which inflation and output are jointly decomposed into unobserved components. This specification has the advantage of averting the exogenous estimation of the output gap, as it is implicitly present in the output equation. In this case, we have a vector of observations that now relies upon two seemingly unrelated time series equations (SUTSE).

The joint specification of inflation and output differs from Harvey (2008) because of the introduction of the seasonal component and, especially, of the expectations term. Our main model now is:

\[
\begin{bmatrix}
\pi_t \\
y_t
\end{bmatrix} = \begin{bmatrix}
\mu_t \\
\mu_t
\end{bmatrix} + \begin{bmatrix}
y_t^\pi \\
y_t^\gamma
\end{bmatrix} + \begin{bmatrix}
\psi_t^\pi \\
\psi_t^\gamma
\end{bmatrix} + \delta E_t(\pi^\pi_{t+1}) + \frac{\sum_{k=1}^l d_k \theta^\pi_{k,t}}{\sum_{k=1}^m f_k \theta^\gamma_{k,t}} + \begin{bmatrix}
\xi_t^\pi \\
\xi_t^\gamma
\end{bmatrix}
\]
where $\theta_{k,t}^{\pi}$ and $\theta_{k,t}^{\nu}$ represent the sets of outliers considered for the inflation and output series, respectively.

The link between the series in the SUTSE approach is generally established by the correlations of errors of one or more components. Following Harvey (2008), it is assumed here that the cycles have the same autocorrelation function and spectrum. In other words, inflation and output cycles are modeled as similar cycles. In algebraic terms, supposing $\psi_t = (\psi_t^{\pi}, \psi_t^{\nu})'$,

$$
\begin{bmatrix}
\psi_t \\
\psi_t^{\nu}
\end{bmatrix} = \rho \begin{bmatrix}
\cos \lambda_c & \sin \lambda_c \\
-\sin \lambda_c & \cos \lambda_c
\end{bmatrix} \otimes I_2 + \begin{bmatrix}
\psi_{t-1} \\
\psi_{t-1}^{\nu}
\end{bmatrix} + \begin{bmatrix}
\kappa_t \\
\kappa_t^{\nu}
\end{bmatrix}, \quad t = 1, \ldots, T \quad (17)
$$

where $\kappa_t$ and $\kappa_t^{\nu}$ are 2 x 1 error vectors, such that $E(\kappa_t, \kappa_t^{\nu}) = \Sigma_\kappa$, and $\Sigma_\kappa$ is a 2 x 2 covariance matrix and $E(\kappa_t, \kappa_t^{\nu}) = 0$.

The series can also be expressed in state-space form, but now the components are vectors. As with univariate estimation, the inflation trend component follows a local level model, as in (13), and the output component conforms to a smooth trend model, as in (8) and (9). Seasonality here is also stochastic, in order to confirm its variability in the inflation series.

The cyclical component of inflation can be broken down into two independent parts, as follows:

$$
\psi_t^{\pi} = \beta \psi_t^{\nu} + \psi_t^{\pi^*} \quad (18)
$$

where $\beta = \frac{\text{Cov}(\psi_t^{\pi}, \psi_t^{\nu})}{\text{Var}(\psi_t^{\nu})} = \frac{\text{Cov}(\kappa_t^{\nu}, \kappa_t^{\nu})}{\text{Var}(\kappa_t^{\nu})}$ and $\psi_t^{\pi^*}$ is a cyclical component specific to inflation.

Thus, the inflation equation may be written as:

$$
\pi_t = \mu_t^{\pi} + \gamma_t + \beta \psi_t^{\nu} + \psi_t^{\pi^*} + \varepsilon_t^{\pi} \quad \varepsilon_t^{\pi} \sim N(0, \sigma^{\pi}_\varepsilon) \quad (19)
$$

Considering that the cycle of the output equation gives a notion about the output gap, as occurred in Section 2, and that disturbances $\kappa_t^{\pi}$ and $\kappa_t^{\nu}$ are perfectly correlated, it is possible to conclude that coefficient $\beta$ corresponds to parameter $\phi$ of the univariate PC, i.e., the slope of the PC. Therefore, from the correlation matrix of cycles, one obtains $\beta = \phi$.

In the bivariate case, three specifications are tested. Again, an approach similar to that of Harvey (2008) is compared with the model built above, in which one includes the future inflation expectations term, as shown in equation (16). Finally, procedures (16) through (19) are repeated considering the IBC-Br for the series of $y_t$.

The relevant goodness-of-fit statistics are now a correlation matrix for the prediction error variance and the log-likelihood. Test statistics already used in the univariate model are also reproduced in Table 3.
3.1 Results

Table 3 shows the bivariate estimation results. The estimates reveal again that the introduction of expectations clearly improves the model’s fit and determination. It is also important to highlight that model V had a low algorithm convergence compared to a very high convergence in model VI. The comparison with model VII is hindered because the samples are different, but a pattern can be detected again in the expectations and gap coefficients.

By looking at Figure 3, it is possible to draw some conclusions about the recent dynamics of inflation and of output in Brazil: The variability of the seasonal effects tends to decrease in the more recent periods of the sample, which once again evinces an effect of consolidation of the inflation targeting system. The less pronounced decrease compared with Figure 2 is also due to the smaller sample size.

<table>
<thead>
<tr>
<th>Model V (Harvey):</th>
<th>Loglik</th>
<th>R²</th>
<th>Q</th>
<th>N</th>
<th>H†</th>
<th>δy</th>
<th>Φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ y_t = \mu_{yt} + \beta y_{t-1} + \gamma_1 y_{t-1} + \sum_{t=1}^{T} \delta y_{t-1} \phi y_{t-1} + \epsilon_t ]</td>
<td>513.57</td>
<td>0.52</td>
<td>18.94</td>
<td>2.57</td>
<td>0.33</td>
<td>-</td>
<td>0.065*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model VI (PIB-BC)</th>
<th>Loglik</th>
<th>R²</th>
<th>Q</th>
<th>N</th>
<th>H†</th>
<th>δy</th>
<th>Φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ y_t = \mu_{yt} + \beta y_{t-1} + \gamma_1 y_{t-1} + \sum_{t=1}^{T} \delta y_{t-1} \phi y_{t-1} + \epsilon_t ]</td>
<td>519.52</td>
<td>0.58</td>
<td>20.63</td>
<td>2.65</td>
<td>0.58</td>
<td>1.05</td>
<td>0.057*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model VII (IBC-Br)</th>
<th>Loglik</th>
<th>R²</th>
<th>Q</th>
<th>N</th>
<th>H†</th>
<th>δy</th>
<th>Φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ y_t = \mu_{yt} + \beta y_{t-1} + \gamma_1 y_{t-1} + \sum_{t=1}^{T} \delta y_{t-1} \phi y_{t-1} + \epsilon_t ]</td>
<td>59.72</td>
<td>0.42</td>
<td>25.66</td>
<td>7.38</td>
<td>0.73</td>
<td>1.22</td>
<td>0.034*</td>
</tr>
</tbody>
</table>

Source: Data obtained by the authors
Notes: *The significance of parameter Φ is not available as this parameter was estimated indirectly, as explained in (18) and (19).
†: Values in square brackets: p-value.
‡: Statistics R²; Q, N and H refer only to the inflation equation.
The interventions considered in the inflation equation in models V and VI, in order of importance, and the respective p-values were:
- Outlier in 2002/11. Model V: 1.41 [0]; Model VI: 1.29 [0]
- Outlier in 2003/6. Model V: -0.66 [6x10^-14]; Model VI: -0.87 [0]
- Outlier in 2003/9. Model V: 0.71 [3x10^-4]; Model VI: 0.67 [3x10^-4]
- Outlier in 2000/8. Model V: 0.61 [0.005]; Model VI: 0.56 [0.008]
In the output equation, level break in 2008/12. Model V: -0.09 [0]; Model VI: -0.09 [0].
In model VII, the resulting interventions were:
- Outlier in 2003/6: -1.16 [0]
- Outlier in 2003/2: -0.74 [9x10^-4]
- Outlier in 2003/9: 0.55 [0.005].

Table 3: Estimation results – bivariate case
As to the GDP, the seasonal effects were reasonably constant in the sample. On the other hand, the cyclical component, which gives some notion about the gap, showed a more erratic behavior, with brisk movements at the end of 2008,$^{17}$ due to the impact of the U.S. real estate crisis. Also, note how the modeling of similar cycles allowed for a contemporaneous pattern in both series close to the U.S. real estate crisis episode.

![Graphs showing economic data](image)

Note: The IBC-Br is constructed based on the value of 100 in 2002. Inflation is expressed in monthly rates.

**Figure 3: Inflation and output (IBC-Br) decomposition – Bivariate model (VII)**

### 4. Extensions

Some analyses were added to the basic models in order to better understand the dynamics of PC components in the Brazilian case. The first one concerns the flattening of the PC, observed in studies for some developed countries. As shown by Kuttner & Robinson (2008), parameter $\phi$ of equation (12), which represents the response of the observed inflation to the output gap, has decreased in empirical analyses of the United States and Australia. To investigate whether the same occurs in Brazil, a variant of model IV was tested, in which the output gap coefficient was allowed to vary over time, i.e., we now have $\phi_t$. In this case, a smoothing spline was used, in which the slope of the PC varies according to:

\[
(\phi_t - \phi_{t-1}) = (\phi_{t-1} - \phi_{t-2}) + u_t \\
\quad u_t \sim N(0, \sigma_{u_t}^2) \tag{20}
\]

$^{17}$ The behavior of IBC-Br in late 2008 suggests a level break in trend, which was not feasible in practice due to restrictions on the algorithm and to the relatively small amount of observations.
The estimation of this new model is carried out with equations (12) through (15) plus (20), which is an additional state equation.

The prediction error variance of this estimation, 0.026, was slightly lower than that of model IV. Results indicate that flattening of the PC has recently been underway in Brazil as well. Figure 4 shows the time evolution of coefficient \( \phi_t \) and the interval of two standard deviations. This result confirms the importance to consider time-varying parameters in PC estimations, as underscored by Sachsida et al (2009). In addition, there is an important policy assumption that the potential cost of disinflation in terms of lost output has increased. On the other hand, increases in economic activity have been accompanied by gradually smaller inflationary pressures. Tombini & Alves (2006) highlight that the mere uncertainty caused by the 2002 electoral crisis would have been strong enough to change the parameters of the reduced-form PC, leading to higher costs of disinflation. The authors also find evidence of reduction in parameter \( \phi_t \).

The modeling proposed in the present study additionally allows assessing the forecasting power of a PC model by comparing the observed inflation with the one calculated through the models built by the Kalman filter, based on the minimization of one-step-ahead forecast errors. Stock & Watson (2008) reviewed works that dealt with forecasting inflation based on some form of PC and observed that these types of forecast are advantageous in some cases. However, Atkeson & Ohanian (2001) advocate that these forecasts tend to be worse than those which are based on simple...
univariate models. Notwithstanding, the widespread use of PC in the literature and the practice with such forecasts require that their forecasting power be evaluated\textsuperscript{18}.

In the present study, the last 12 observations were excluded and the one-step-ahead inflation forecast was estimated for the period between April 2009 and March 2010. The mean squared error values for each model are shown in Table 4.

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>Naive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean squared error:</td>
<td>0.0258</td>
<td>0.0235</td>
<td>0.0215</td>
<td>0.0269</td>
<td>0.0222</td>
<td>0.0208</td>
<td>0.0184</td>
<td>0.0296</td>
</tr>
</tbody>
</table>

\textit{Source:} Data collected by the authors

\textbf{Table 4: Mean squared forecast error}

Note that the model without expectations had a lower forecasting power than the model with expectations, corroborating again the argument of the present study. This occurred both in the univariate and bivariate cases. In the univariate specification, the gap extracted from IBC-Br was not very successful, but in the bivariate case, it yielded the lowest mean squared error among all estimations.

It should be underscored that the forecasting power increases in all cases when a multivariate specification is used. Finally, the mean squared error of a naive inflation model was calculated. In such a model, expected inflation value is forecasted by its current value, i.e., $E_{t-1}\pi_t = \hat{\pi}_t = \pi_{t-1}$. All analyzed cases of PC outperformed this specification.

5. Conclusions

Given the clear-cut empirical difficulties surrounding the PC and in order to fill a gap in its investigation in Brazil, the present study assessed inflation dynamics using an estimation with unobserved components for the Brazilian economy. By modifying Harvey’s (2008) approach, introducing an inflation expectation term in the PC, the model manages to parsimoniously reproduce some stylized facts about the relation between inflation and output gap, at least when the IBC-Br index is used. With the additional advantage of the graphical result, which allows a more direct economic interpretation of the components, we highlight the variability of the seasonal component of inflation, even with a sample of relatively few years. The relative reduction in this

\textsuperscript{18} Araujo & Guillen (2008) test the forecasting power of different PC specifications according to the gap measure used and conclude that the best performance was obtained by the gap extracted by the multivariate method of unobserved components.
variability in the past years suggests that the inflation targeting system has contributed to reducing not only the inflation rates, but also their volatility within each year.

The detrended output gap built by the PIB-BC series and the ICU deviation series did not yield good statistical results for the analyzed PC, even though positive coefficients were always found. In the case of an output gap extracted from the IBC-Br series, the result was clearly better, showing that this index, yet not used in academic works, may be of great value in monitoring Brazilian monetary policy. Such success indicates that the output gap can also be representative in the Brazilian inflation dynamics, depending on the index used. Previous studies have normally used quarterly GDP series or the BCB’s monthly series, as the one used here, in model II. In the former case, the number of observations is too small and, in the latter, the extrapolation of quarterly values to monthly ones is unlikely to allow capturing the output dynamics. Thus, the output gap results of the IBC-Br series may again strengthen the crucial relation of the PC for the Brazilian case.

The analysis of the PC slope, represented by parameter $\phi_t$, is another important aspect. Models that considered the fixed coefficient did not provide very accurate values for the analyzed sample. On the other hand, the analysis of the first extension, despite its poorly accurate results, indicates that flattening of the PC occurs in Brazil similarly to what is observed in developed countries, as reported by Kuttner & Robinson (2008). An important implication is that PC estimations for Brazil for larger periods should take this movement into consideration, otherwise the output parameter will be overestimated at the end of the sampled period. This reduction in the impact of deviations of output from its real level on inflation means, ceteris paribus, that increases in economic activity, would not produce so much inflationary pressure as they used to. However, the costs of disinflation, in terms of lost output, would tend to increase in this scenario.

Finally, the forecasting power of different models was tested against a simple forecasting model. All models could outperform it in terms of squared forecast errors for the last 12 months of the sample.

Some issues, which were not dealt with here and that could be subject of investigation of future research, include the following: comparison of the performance of the output gap with that of other measures, such as unit labor cost or deviation from the natural rate of unemployment; another approach that considers different dynamics of free and administered prices, and even possible distinctions between tradable and nontradable goods, which concern the exchange rate influence; and finally, similar estimations for other countries.

As more data become available, it is likely that quarterly samples will have a higher forecasting power and will be successfully used in similar estimations. In the meantime, the study showed that IBC-Br can be an important tool for the formulation of monetary policy in Brazil.

References


