REAL (EFFECTIVE) EXCHANGE RATE IN URUGUAY:
A PERIODIC COINTEGRATION APPROACH
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Abstract
When the seasonal component of a particular time series is treated as if it were a mere deterministic phenomenon instead of a stochastic one, it may lead to inconsistent estimations, statistical inference errors and policy biases. This issue is addressed in this paper focusing on the real effective exchange rate in Uruguay for the 1983:1-2006:4 period.

Keywords: real effective exchange rate, periodic cointegration, Uruguay.

JEL: F31, C22, N16

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1 The opinions expressed herein are those of the author’s and do not reflect necessarily those of the Central Bank of Uruguay.
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I. INTRODUCTION

It is not a trivial question to know whether the real effective exchange rate (REER) is stationary or not because, among other reasons, the effects of random shocks could be temporary or last forever. This feature is a crucial one for policy purposes when it is taken into account the role played by the REER in the resource allocation mechanism and the profitability among different economic sectors. There are a number of reasons why a particular time series could resemble nonstationary, from the happening of structural breaks to the mistreatment of its seasonal pattern. Sometimes, structural breaks affect the long-run relationship among the cointegrated time series involved2 but the short-run specification stands still. And if a stable functional specification of the REER could be found taking into account those breaks, the effects of any shock are expected to fade away in the long run. The same reasoning is true for the incorrect approach to the actual nature of the seasonal pattern of the time series tied up by a common performance in the long-run, that is to say, time series that are cointegrated. Different long-run elasticities (preferences) and different speeds of adjustment towards long-run equilibria could be misunderstood as instability signals instead of just logical features stemming from the combination of the stochastic seasonal patterns of periodically cointegrated time series.

Usually, the seasonal component of many economic time series is treated as if it were a totally deterministic phenomenon which implies that it is unchangeable and could be perfectly forecasted. But for many time series seasonality can be stochastic and sometimes “Spring” may become “Summer” and others “Autumn”. In addition, cointegrated time series could share stable long-run relationships but could have variable seasonal patterns that may not coincide in the short run. Models that disregard the true nature of the seasonal component could lead to inconsistent estimations, statistical inference errors and economic policy biases.

In this paper the REER for Uruguay is estimated for the 1983:1-2006:4 period using a periodic cointegration approach. In that way, cointegrated time series that have stochastic seasonal patterns could render stable long-run preferences that nevertheless change with the season (quarter) and different speeds of adjustment to long-run equilibrium relationships according to the season (quarter).

The plan of this paper is as follows. In the next session, the analytical framework is presented and statistical properties of the time series involved are analyzed. Then, periodical cointegration approach is briefly explained and performed. Finally, some concluding remarks are drawn.

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2 It is well-known that the cointegration modelling approach is more prone to suffer from structural breaks in the long-run relationships.
II. ANALYTIC FRAMEWORK AND TIME SERIES INVOLVED

In this paper, I will take a definition of external real exchange rate\(^3\) and concentrate on trying to describe its data generating process through a periodic cointegration error-correction approach\(^4\).

II.1 Definition of RER

As the real exchange rate (RER) is just a relative price between two consumption bundles, there can be as many definitions of RER as types of bundles considered. For instance, it could be defined as the relationship between the domestic consumer price index \((P)\) and the foreign consumer price index \((P^*)\), both expressed in a common currency:

\[
RER = \frac{P^*}{E} \cdot \frac{P}{P^*} = \frac{E}{P} \cdot P^*
\]

where \(E\) is nominal exchange rate (domestic currency per unit of foreign currency). This relationship can be expressed as a bilateral or a multilateral one and the latter is referred to as real effective exchange rate. In REER, each bilateral relationship expressed in a common currency such as American dollars (REER\(^{cc}\)) is weighed according to the relative importance of each country in the domestic country international trade, \(\alpha_i\):

\[
REER = \sum_{i=1}^{m} \alpha_i \cdot REER_i^{cc}
\]

where \(m\) is the number of commercial partners in the domestic country international trade.

The Purchasing Power Parity (PPP) theory points out that international trade arbitrage guarantees that the price of the same bundle in different countries must be the same expressed in a common currency. That leads to a constant unit value for the RER. In its relative version, PPP establishes that through international trade arbitrage the rate of change of domestic and foreign prices expressed in the same currency must be the same. This implies that RER should be constant. Nevertheless, should the RER be stationary, transitory deviations could be allowed in the short run for the RER should converge to its fundamentals in the long run. According to the literature, there are a number of fundamentals for the REER: productivity, consumption, fiscal policy, capital flows, foreign and domestic interest rates, terms of trade, openness degree.

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\(^3\) *External* real exchange rate measures the purchasing power of domestic currency by comparing the same bundle of goods between two economies; *internal* real exchange rate measures the relative price of tradable goods in terms of the price of nontradable ones both produced in the domestic economy. The former is better in order to evaluate the impact of nominal shocks in the economy while the latter is particularly useful to analyze the effects of shocks on tradable good sector productivity. See Hinkle and Montiel (1999).

\(^4\) It is more appropriate to address *error-correction* models as *equilibrium-correction* ones. See Ahumada (2006).
II.2 Theoretical framework


The real exchange rate is perceived as such a crucial ratio that guarantees simultaneous balance of both domestic and external sectors. That is to say, it is assumed that there is an underlying relationship between absorption and RER so as to balance both of them. But the sign of it defers depending on which equilibrium is to be restored. If the economy is in equilibrium, an excess demand of nontradable goods (an increase in absorption) needs to be compensated by a negative excess demand of tradable goods through a real appreciation. So, the relationship between absorption and RER is negative. If the economy is in equilibrium, an excess demand of nontradable goods leads to a current account deficit which requires a real depreciation to restore equilibrium. So, the relationship between absorption and RER is positive.

The obvious way to follow this research is to focus on the fundamentals of absorption in order to try to capture an assumed stable long-run relationship among RER and those ones.

II.2.1 Productivity (\(\pi\))

Balassa (1964) argues that productivity growth is different not only for each economic sector but also for each economy and that it is in fact higher in the tradable good sectors. The Balassa-Samuelson effect points out that a rise in productivity appreciates RER. On the one hand, a rise in the tradable sector relative productivity would increase tradable output and decrease tradable good prices. On the other, it would increase labor demand and reallocate labor inputs from nontradable good
sectors to tradable ones which would increase wages economy-wide. This implies a negative supply shock to the nontradable good sector which requires a change (rise) in its relative price in order to restore long-run equilibrium. Those two effects would make RER appreciate.

II.2.2 Consumption (CT, CN)
A rise in consumption of tradable goods would deteriorate the current account and would need a real depreciation to restore equilibrium. A rise in consumption of nontradable goods would cause an excess demand, a rise in nontradable good prices and a real appreciation. There is compelling evidence that points out that Government consumption (G) is more intensive in nontradable goods than private consumption is.

II.2.3 Fiscal policy
As long as taxes are not distortionary, it seems that only the level and composition of fiscal expenditure are relevant in the long-run determination of RER disregarding the sources of financing that budget. Nevertheless, Ricardian equivalence may not hold when a particular indebtedness policy could generate a positive real effect and a real appreciation in the short run.

II.2.4 Capital flows (k)
Capital flows are regarded as a loosen of the economy budget constraint which increases absorption levels at least in the short run. Furthermore, nontradable excess demand would rise its price and appreciate RER. But the actual effects of capital flows on RER depend on the type of flows received by the economy. If those capital flows are permanent, RER would appreciate; if those capital flows are only transitory and must be reimbursed later plus their corresponding interests, RER could depreciate in the long run. (See Morrissey et al, 2004).

II.2.5 Foreign, domestic and net real interest rates ($r^*, r, in$)
Another way of loosening the foreign budget constraint is a drop in international real interest rates. That would mean a higher commercial deficit with a real appreciation. In the domestic sector, a drop in international real interest rates would push down investment costs making new projects rewarding, fostering nontradable demand and appreciating RER.

Mac Donald and Ricci (2003) suggest that real interest rate differentials could capture a Balassa-Samuelson residual left out because of measurements problems in the productivity proxy used in the calculus. In fact, real domestic interest rate is the price of capital within the economy and a rise in that with respect to the international one, could be interpreted as a capital productivity rise which would lead to a real appreciation.

II.2.6 Terms of trade (TOT)
A rise in the terms of trade is expected to appreciate RER. A higher exportable price relative to the importable one attracts resources to the tradable sector similar to a negative supply shock to the nontradable sector. With a positive wealth effect caused by higher TOT, an excess demand for nontradable goods appears which appreciates RER. There is a second order effect that would depreciate RER stemming from the
fact that the relative reduction of importable prices could reduce nontradable demand.

In commodity-export economies such as Uruguay the effects of changes of terms of trade are expected to be significant (Dutch disease).

II.2.7 Openness degree (OD)

An increase in the degree of openness of the domestic economy would cause commercial account deficit through an increase in the importable demand and would require RER to depreciate to restore equilibrium. Internally, two shocks would appear: a positive supply one, because of the cheapening of imported inputs and a negative demand one, because of consumption substitution from nontradable goods to imported ones. Both shocks lead to a reduction in nontradable prices and a real depreciation.

The underlying relationship among REER and its fundamentals to be revised in the course of this investigation can be expressed as:

$\text{(3) } \text{REER} = f(\pi^-, CT^+, CN^-, G^-, K^-, r^{**}, r^-, in, TOT^-, OD^{-})$

where the sign on the upper right of each variable reflects the expected marginal response of REER to marginal changes in each fundamental. The variables that will be actually considered will depend on data availability and will be the remaining ones after cointegration tests are performed:

$\text{(3)'} \text{REER} = f(G^-, in^-, TOT^-)$

III. MODELLING APPROACH

III.1 Periodic cointegration

Periodic cointegration models allow that both long-run parameters and adjustment coefficient vary with the season. It can be the case that there is an adjustment towards the LR relationship in each season (totally cointegrated) or that there is no adjustment in some of the seasons (partially cointegrated).

The final specification should be:

$\Delta y_t = \lambda_s (y_{t-s} - \theta_t z_{t-s}) + \sum_{i=1}^{p-1} \gamma_i \Delta y_{t-i} + \sum_{s=1}^{k} \beta_i \Delta_s z_{t-s}$

where $y_t$ is the dependent variable (REER), $z_t$ is an independent variable vector, $\lambda_s$ reflects adjustment parameters in season $s$ and $\theta_s$ is the long-run parameter vector in season $s$.

In order to perform a periodic cointegration estimation, it is imperative to investigate the existence of unit roots in the different seasons $s$ for each time series. Franses (1996) proposes to start from analyzing the periodic autocorrelation of each time series, then write it down as a vector of quarters representation (VQ) and finally test whether the corresponding characteristic equation has a unit root (UR). If only one UR is found, and the others lie outside the unit circle then, it can be said that there is a common trend and three cointegrating relationships. For the explanatory variables Franses advises to use HEGY test.
In a second step, it should be analyzed the possibility of more than one cointegrating relationships in a periodic ECM, where all variables are transformed into (periodically) stationary time series through the $\Delta_4$ filter. Next, it should be tested whether there is partial or total periodic cointegration (Wald cointegration tests) between REER and its fundamentals. If the null hypothesis of no cointegration is rejected, then it must be tested whether long-run elasticities are the same (including long-run intercepts) and whether adjustment parameters are the same. Finally, weak exogeneity tests must be performed in order to rule out alternative specifications.

III.2 Time series identification

In this section, I will characterize the time series involved in this study focusing mainly on the seasonal component of each one.

III.2.1 Unit roots

Hylleberg, Engle, Granger and Yoo (HEGY, 1990) propose a test to determine whether a univariate time series has unit roots both at seasonal and at zero frequencies (long run). That test follows the standard Dickey-Fuller approach, with transformed series to cover special cases.

HEGY use the factorization derived from the known Box and Jenkins seasonal filter for quarterly data:

$$(1 - B^4) = (1 - B)(1 + B)(1 + B^2),$$

where $B$ is the lag operator. If the time series $x_t$ has unit roots at all seasonal frequencies $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ of one cycle ($2\pi$), each one of the expressions $(x_t - x_{t-1})$, $(x_t + x_{t-1})$, $(x_t + x_{t+2})$ and $(x_t - x_{t-2})$ is non stationary. Assuming that the data are generating by a general autoregressive process, $x_t = \varepsilon_t$, an expansion around the values $\theta_k = +1, -1, +i, -i$ defines a procedure for testing the integration order of the series through an extension of the zero-frequency Dickey-Fuller procedure. The test is based on the auxiliary equation:

$$t_t = (1 - B^4)x_t = \pi_1 y_{3t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \varepsilon_t,$$

where

$y_{3t} = (1 + B + B^2 + B^3)x_t$ is the observed time series adjusted by the seasonal unit roots in $\theta = \frac{3}{4}, \frac{1}{2}, \frac{3}{4}$

$y_{2t} = -(1 - B + B^3)x_t$ is the observed time series adjusted by the seasonal unit roots in $\theta = 0, \frac{1}{4}, \frac{3}{4}$

$y_{3t} = -(1 - B^2)x_t$ is the observed time series adjusted by the seasonal unit roots in $\theta = 0, \frac{1}{2}$.

Tests for UR at frequencies $0$, $\frac{1}{2}$, $\frac{3}{4}$ are based on “t-values” for $\pi_1$ and $\pi_2$ which are distributed as a Dickey-Fuller distribution and an F test $\pi_3 \cap \pi_4$; when $\pi_4 = 0$, it is used “t-value” for $\pi_3$. 
## Chart 1 – SEASONAL INTEGRATION TESTS (HEGY)
Sample: 1983.1-2006.4

<table>
<thead>
<tr>
<th>Time series</th>
<th>Auxiliary regression</th>
<th>Values in the sample</th>
<th>Deterministic part</th>
<th>Dependent variable lags</th>
<th>“t”</th>
<th>“t”</th>
<th>“t”</th>
<th>“t”</th>
<th>“F”</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>π₁</td>
<td>π₂</td>
<td>π₃</td>
<td>π₄</td>
<td>π₃ ∩ π₄</td>
</tr>
<tr>
<td>REER</td>
<td>I</td>
<td></td>
<td>1, 2, 3</td>
<td></td>
<td>-2.88</td>
<td>-5.01**</td>
<td>-4.06**</td>
<td>-2.04</td>
<td>10.32**</td>
</tr>
<tr>
<td>Productivity</td>
<td>I</td>
<td></td>
<td>1</td>
<td></td>
<td>-3.33**</td>
<td>-0.36</td>
<td>-1.92*</td>
<td>-0.31</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td>I, SD₁,₂,₃</td>
<td></td>
<td>-</td>
<td></td>
<td>-2.50</td>
<td>-5.07**</td>
<td>-7.43**</td>
<td>-4.47**</td>
<td>57.00**</td>
</tr>
<tr>
<td>Gov Consumption</td>
<td>I</td>
<td></td>
<td>1</td>
<td></td>
<td>-2.19</td>
<td>-2.09*</td>
<td>-2.67**</td>
<td>0.41</td>
<td>3.66*</td>
</tr>
<tr>
<td></td>
<td>I, SD₁</td>
<td></td>
<td></td>
<td></td>
<td>-1.77</td>
<td>-4.48**</td>
<td>-5.12**</td>
<td>-0.61</td>
<td>13.15**</td>
</tr>
<tr>
<td>Foreign int. rate</td>
<td>I, T</td>
<td></td>
<td>-</td>
<td></td>
<td>-3.24</td>
<td>-7.52**</td>
<td>-5.35**</td>
<td>-6.55**</td>
<td>52.03**</td>
</tr>
<tr>
<td>Net interest rate</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td>-2.08**</td>
<td>-6.05**</td>
<td>-6.94**</td>
<td>-4.74**</td>
<td>54.51**</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>I</td>
<td></td>
<td>1, 2, 3</td>
<td></td>
<td>-2.62</td>
<td>-1.68</td>
<td>-2.51</td>
<td>-0.67</td>
<td>3.40*</td>
</tr>
<tr>
<td></td>
<td>I, SD₁</td>
<td></td>
<td>1, 2, 3</td>
<td></td>
<td>-2.19</td>
<td>-2.74</td>
<td>-3.38</td>
<td>-0.68</td>
<td>5.83</td>
</tr>
</tbody>
</table>

### Critical values for REER

- **w/o seasonal dummies, at 5%**: -2.88, -1.95, -1.90, -1.68, 3.08
- **1%**: -3.47, -2.61, -2.61, -2.38, 4.77

### Critical values for π

- **w/o seasonal dummies, at 5%**: -2.88, -1.95, -1.90, -1.68, 3.08
- **1%**: -3.47, -2.61, -2.61, -2.38, 4.77
- **w/seasonal dummies, at 5%**: -2.95, -2.94, -3.44, -1.96, 6.57
- **1%**: -3.55, -3.60, -4.06, -2.78, 8.74

### Critical values for G

- **w/o seasonal dummies, at 5%**: -2.88, -1.95, -1.90, -1.68, 3.08
- **1%**: -3.47, -2.61, -2.61, -2.38, 4.77
- **w/seasonal dummies, at 5%**: -2.95, -2.94, -3.44, -1.96, 6.57
- **1%**: -3.55, -3.60, -4.06, -2.78, 8.74

### Critical values for r*

- **w/o seasonal dummies, at 5%**: -3.47, -1.94, -1.89, -1.65, 2.98
- **1%**: -4.07, -2.58, -2.56, -2.38, 4.70

### Critical values for r

- **w/o seasonal dummies, at 5%**: -1.97, -1.92, -1.90, -1.68, 3.12
- **1%**: -2.60, -2.61, -2.55, -2.43, 4.89
- **w/seasonal dummies, at 5%**: -2.95, -2.94, -3.44, -1.96, 6.57
- **1%**: -3.55, -3.60, -4.06, -2.78, 8.74

### Critical values for TOT

- **w/o seasonal dummies, at 5%**: -2.88, -1.95, -1.90, -1.68, 3.08
- **1%**: -3.47, -2.61, -2.61, -2.38, 4.77
- **w/seasonal dummies, at 5%**: -2.95, -2.94, -3.44, -1.96, 6.57
- **1%**: -3.55, -3.60, -4.06, -2.78, 8.74

**Notes:**
1. The time series are: REER = Real effective exchange rate, π = productivity, G = government consumption as a share of GDP, r* = foreign interest rate, in = net interest rate, TOT = terms of trade.
2. The auxiliary regression can be augmented by its deterministic part: intercept (I), trend (T), deterministic seasonal dummies (SD₁, i=1, 2, 3, 4) or by lags in the dependent variable.
3. Critical values correspond to “t-values” of Dickey-Fuller distribution and to “F-values” for 5%. They were taken from HEGY (1990) and Fuller (1976) for t₁ and t₂ and Dickey, Hasza and Fuller (1984) for t₃.
4. (*) and (**) indicates rejection of unit root hypothesis at 5% and 1%, respectively.
5. Different sample sizes were used for each regression, depending on the number of lags needed to obtain uncorrelated errors.
According to the results presented in Chart 1, all time series analyzed are integrated of order one at zero frequency at 1%\%; terms of trade has also unit roots at biannual and annual frequencies and REER has a unit root at \( \frac{3}{4} \) frequency. The tests performed have difficulty in separating a unit root at the \( \frac{1}{4} \) and \( \frac{3}{4} \) frequencies from a seasonal deterministic pattern. When standard HEGY tests are applied without seasonal dummy variables, productivity, government consumption and terms of trade seem to be integrated of order one at all frequencies; on the other hand, when deterministic seasonal variables are added, the joint test rejects nonstationarity of productivity and government consumption at \( \frac{1}{4} \) and \( \frac{3}{4} \) frequencies (at 1\%). Foreign interest rate and net interest rate have only one unit root each at zero frequency (long run).

**III.2.2 Periodic autoregressive representation**

A periodic autoregressive model of order \( \rho \), \( \text{PAR}(\rho) \), is expressed as:

\[
y_t = \mu_t + \phi_{1s} y_{t-1} + ... + \phi_{\rho s} y_{t-\rho} + \varepsilon_t
\]

where \( \mu_t \) is an interception term that varies with the season \( s \) (quarter), and \( \phi_{1s}, ..., \phi_{\rho s} \) are autoregressive parameters up to order \( \rho \) that could change with the season \( s \), \( s = 1, 2, 3, 4 \). It is assumed that \( \varepsilon_t \) is a standard white noise process with constant variance \( \sigma^2 \), although that assumption could be relaxed to allow seasonal variance \( \sigma_s^2 \).

A quarterly time series is **periodically integrated of order one**, PI(1), when it is needed a filter \((1-\alpha_s B)\) to remove the stochastic trend from it; \( \alpha_s \) the seasonally-variable parameters, verify that \( \alpha_1 \alpha_2 \alpha_3 \alpha_4 = 1 \) and \( \alpha_s \neq \alpha \) for all \( s = 1, 2, 3, 4 \). This definition indicates that PI nests the usual \((1-B)\) filter and the \((1+B)\) filter too, which corresponds to the seasonal unit root at the biannual frequency. As Franses (1996) points out, this suggests that a useful strategy in the case of three cointegrating relations in \( Y_T \) is first to check whether \( \alpha_1 \alpha_2 \alpha_3 \alpha_4 = 1 \) and then to check whether \( \alpha_1 \alpha_2 \alpha_3 \alpha_4 = -1 \).

Franses (1996) advises first to choose the order of periodic autocorrelation, next to test periodic variation of the parameters and finally to test for unit roots. He says,

“\... Since a periodic autoregression allows the AR parameters to take different values in different seasons, it seems natural to allow for the possibility of periodically varying differencing filter which can be used to remove the stochastic trend. A specific periodic differencing filter corresponds to the notion of periodic integration (PI)’’…”The implication of periodic integration is that the stochastic trend and the seasonal fluctuations are not independent, in the sense that accumulation of shocks can change the seasonal pattern and that the time series cannot be decomposed in two strictly separate trend and seasonal components.”

There are two possible ways of choosing \( \rho \), the order of periodic autocorrelation. According to one of them, the analyst can concentrate on the residuals coming from a non periodic autoregressive model; according to the other one, the analyst can estimate a \( \text{PAR}(\rho) \) model, choose \( \rho \) using conventional criteria for model selection and then check whether there is periodic variation in the autoregressive parameters.

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\(^5\) The time series \( y_t \) is expressed in its vector of quarters representation.
Usually, time series residuals give some guidelines of their underlying data generating process. Autocorrelation and time-varying variance of the residuals may be signs of different coefficients for each season (quarter) in an autoregressive representation of a particular time series. In Chart 2 it is presented a summary of one of the procedures used in this paper to determine the number of periodic autoregressive parameters for the time series assumed to be the real fundamentals of the REER for Uruguay.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Time series</th>
<th>k</th>
<th>Diagnostic statistic values</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>REER</td>
<td>Δ₁ REERₜ</td>
<td>5</td>
<td>0.00(1.00) 0.00(1.00) 1.46(0.22) 2.20(0.07) 0.45(0.71)</td>
<td>1</td>
</tr>
<tr>
<td>Productivity</td>
<td>Δ₁ πₜ</td>
<td>5</td>
<td>1.65(0.20) 3.29(0.02) 0.56(0.69) 0.94(0.49) 1.05(0.37)</td>
<td>1</td>
</tr>
<tr>
<td>Gov. Consumption</td>
<td>Δ₁ Gₜ</td>
<td>4</td>
<td>7.53(0.01) 4.14(0.004) 0.67(0.61) 0.68(0.71) 0.43(0.73)</td>
<td>1</td>
</tr>
<tr>
<td>Foreign int.rate</td>
<td>Δ₁ rₜ</td>
<td>5</td>
<td>6.24&quot;(0.01) 2.59&quot;(0.04) 4.20&quot;(0.00) 4.32&quot;(0.00) 1.71(0.17)</td>
<td>2</td>
</tr>
<tr>
<td>Net interest.rate</td>
<td>Δ₁ rₜ</td>
<td>10</td>
<td>5.52&quot;(0.02) 1.33(0.27) 1.45(0.22) 1.40(0.21) 0.47(0.71)</td>
<td>1</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>Δ₁ TOTₜ</td>
<td>4</td>
<td>10.87&quot;(0.01) 3.54&quot;(0.01) 0.93(0.45) 1.91(0.07) 1.18(0.32)</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes:
1. All time series are transformed according to seasonal and non-seasonal UR tests results, see Chart 1.
2. Diagnostic statistics are: residual autocorrelation of order 1 and order 1-4 (4 lags), $F_{AR,1-4}$; periodic residual autocorrelation of order 1 and order 1-2 (2 lags), $F_{Piar,1-2}$ and seasonal heteroskedasticity, $F_{SH}$.
3. The periodic residual autocorrelation test is based on the auxiliary regression:
   $$\hat{v}_t = \sum_{i=1}^{m} \psi_1 v_{t-i} + \sum_{i=1}^{4} \psi_{1-i} v_{t-i} + \cdots + \psi_{m} v_{t-m} + u_t$$
   applied on the residuals $v_t$ that come from an AR(k) model on the $x_t$ time series, where $x_t = \Delta z_t$. Under the null hypothesis of no periodic autocorrelation of order $m$, $\psi_1 = \cdots = \psi_m = 0$, the F statistic follows a standard F distribution asymptotically with $(4m, n-k-4m)$ degrees of freedom. Critical values for $m=1$ are 2.46 (5%) and 3.51 (1%) and for $m=2$ are 2.03 (5%) and 2.69 (1%).
4. The seasonal heteroskedasticity test is based on the auxiliary regression:
   $$\hat{v}_t^2 = w_1 + w_2 D_{1t} + w_3 D_{2t} + w_4 D_{3t} + \lambda_t$$
   Under the null hypothesis of seasonal homoskedasticity, $w_1 = w_2 = w_3 = 0$, and the corresponding F statistic follows a F standard distribution with $(3, n-k)$ degrees of freedom. Critical values are: 3.98 (5%), 3.98 (1%).
5. $k$ stands for the order of the autoregressive process AR(k) while $\rho$ stands for the order of the periodic autoregressive process $PAR(\rho)$.
   * Significant at 5%
   ** Significant at 1%

Another way of choosing the order of periodic autocorrelation is to adjust an equation such as:

$$x_t = \sum_{i=1}^{4} \mu_x D_{1t} + \sum_{i=1}^{4} \phi_{1-i} D_{is} x_{t-i} + \cdots + \sum_{i=1}^{4} \phi_{p-i} D_{ps} x_{t-p}$$
with \( x_t \) the original time series and \( s = 1, 2, 3, 4 \). Standard AIC (Akaike Information Criteria) and SC (Schwarz Criteria) are used and it taken into account that parameters are estimated for different seasons. It is possible to perform an “F test” to analyze autoregressive-parameter significance of order greater than \( \rho \), \( F_{\text{PAR}} \). Besides, residuals could be tested in order to investigate periodic heteroskedasticity. Franses (1996) advises to use SC to choose the order of \( \rho \) as long as it could not be possible to reject the null hypothesis of \( \Phi_{\rho+1,s} = 0 \).

### Chart 3- ORDER SELECTION IN PERIODIC AUTOREGRESSIVE MODELS, \( \text{PAR}(\rho) \)

Tests based on estimated \( \text{PAR}(\rho) \) models

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \rho )</th>
<th>( F_{\text{AR}, 1-1} )</th>
<th>( F_{\text{AR}, 1-4} )</th>
<th>( F_{\text{ARCH}, 1-1} )</th>
<th>( F_{\text{ARCH}, 1-4} )</th>
<th>( J_B )</th>
<th>( F_{\text{SH}} )</th>
<th>( F_{\text{PAR}, 1-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>REER</td>
<td>1</td>
<td>0.00(1.00)</td>
<td>0.00(1.00)</td>
<td>0.01(0.89)</td>
<td>0.06(0.99)</td>
<td>2316.7** (0.00)</td>
<td>0.77(0.52)</td>
<td>1.92(0.11)</td>
</tr>
<tr>
<td>Productivity</td>
<td>1</td>
<td>0.00(1.00)</td>
<td>0.00(1.00)</td>
<td>0.32(0.57)</td>
<td>0.52(0.72)</td>
<td>25.87** (0.00)</td>
<td>1.85(0.19)</td>
<td>0.60(0.66)</td>
</tr>
<tr>
<td>Gov.Cons.</td>
<td>6</td>
<td>0.00(1.00)</td>
<td>0.00(1.00)</td>
<td>2.33(0.13)</td>
<td>0.76(0.57)</td>
<td>6.18(0.05)</td>
<td>1.46(0.23)</td>
<td>0.22(0.93)</td>
</tr>
<tr>
<td>Foreign int.</td>
<td>1</td>
<td>0.00(1.00)</td>
<td>0.00(1.00)</td>
<td>8.24** (0.00)</td>
<td>7.16** (0.00)</td>
<td>33.80** (0.00)</td>
<td>3.69** (0.00)</td>
<td>9.06** (0.00)</td>
</tr>
<tr>
<td>Net int. rate</td>
<td>1</td>
<td>0.00(1.00)</td>
<td>0.00(1.00)</td>
<td>23.38** (0.00)</td>
<td>5.57** (0.00)</td>
<td>1.33(0.51)</td>
<td>0.25(0.86)</td>
<td>1.44(0.23)</td>
</tr>
<tr>
<td>Terms of tr.</td>
<td>2</td>
<td>0.00(1.00)</td>
<td>0.00(1.00)</td>
<td>2.03(0.16)</td>
<td>3.13** (0.02)</td>
<td>30.74** (0.00)</td>
<td>6.17** (0.00)</td>
<td>2.51** (0.00)</td>
</tr>
</tbody>
</table>

Notes:

1. The order of \( \rho \) was chosen according to Schwarz Criteria:
   \[ SC(\rho) = n \log \sigma^2 + 4 \log n \] , with \( n = 4N \), the whole sample.
2. The adjustment for REER (standard errors between brackets) is:
   \[
   \begin{aligned}
   \text{REER}_t & = 0.9984 D_{t,1} \text{REER}_{t-1} + 0.9939 D_{t,2} \text{REER}_{t-1} + 0.9998 D_{t,3} \text{REER}_{t-1} + \\
   & \quad + 1.0060 D_{t,4} \text{REER}_{t-1} + \varepsilon_t \\
   \end{aligned}
   \]
   (253.11) (256.25) (256.22)
3. The adjustment for productivity (standard errors between parenthesis) is:
   \[
   \begin{aligned}
   \pi_t = -0.12 D_{t,1} + 0.10 D_{t,2} + 0.8079 D_{t,3} \pi_{t-1} + 0.9345 D_t \pi_{t-1} + 1.0268 D_t \pi_{t-1} + 0.8448 D_t \pi_{t-1} + \varepsilon_t \\
   \end{aligned}
   \]
   (−19.14) (9.07) (8.96) (19.76) (20.85) (10.41)
4. The adjustment for government consumption (standard errors between parenthesis) is:
   \[
   \begin{aligned}
   G_t = 1.011408 D_t G_{t-1} + 0.9959 D_t G_{t-1} + 0.9994 D_t G_{t-1} + 1.0083 D_{t,4} \ G_{t-1} + \varepsilon_t \\
   \end{aligned}
   \]
   (182.02) (164.42) (174.95) (179.84)
5. The adjustment for foreign interest rate (standard errors between parenthesis) is:
   \[
   \begin{aligned}
   i^*_t = 1.0024 D_{t,1} i^*_{t-1} + 1.0073 D_t i^*_{t-1} + 0.9992 D_t i^*_{t-1} + 0.9703 i^*_{t-1} + \varepsilon_t \\
   \end{aligned}
   \]
   (50.71) (51.59) (51.67) (50.31)
6. The adjustment for net interest rate (standard errors between parenthesis) is:
   \[
   \begin{aligned}
   in_t = 1.0339 D_t in_{t-1} + 0.9728 D_t in_{t-1} + 0.9935 D_t in_{t-1} + 0.9508 D_t in_{t-1} + \varepsilon_t \\
   \end{aligned}
   \]
   (44.39) (44.75) (44.76) (44.76)
7. The adjustment for terms of trade (standard errors between parenthesis) is:
   \[
   \begin{aligned}
   TOT_t = 1.1068 D_t TOT_{t-1} + 0.9140 D_t TOT_{t-1} + 0.9836 D_t TOT_{t-1} + 1.0173 D_t TOT_{t-1} + \varepsilon_t \\
   \end{aligned}
   \]
   (80.83) (75.10) (74.01) (74.29)
8. Columns three to eight, test results on residuals of \( \text{PAR}(\rho) \) models are presented:
   \( F_{\text{AR}} \) (residual autocorrelation), \( F_{\text{ARCH}} \) (heteroskedasticity), \( J_B \) (normality), \( F_{\text{SH}} \) (seasonal heteroskedasticity).
9. Periodic variation of autoregressive parameters is tested by \( H_0: \Phi_{s,s} = \phi_s \) for \( s = 1, 2, 3, 4 \)

---

6 Franses (1996) points out that it is not necessary to take first differences of the time series to remove stochastic trends when periodicity is being analyzed.
and \( j=1,2,\ldots, \rho \) and the statistic \( F_{\text{PAR}} \) is distributed as a standard \( F(3\rho, n-(4+4\rho)) \).

- Critical values are 2.70(5\%) and 3.98(1\%) for \( \rho=1 \) and 2.21(\%) and 3.04(1\%) for \( \rho=2 \).
- \* Significant at 5\% 
- \** Significant at 1\% 

According to the results presented in Chart 3, all time series could be described as periodic autoregressive time series of order one, except for government consumption as a share of GDP and terms of trade, which appear to be periodic autoregressive time series of order two\(^7\). Diagnostic analyses performed to detect residual autocorrelation, ARCH patterns, non normality and seasonal heteroskedasticity suggest that PAR(1) and PAR(2) models give a better description of the data.

The following step is to test for unit roots. We first write each PAR(1) process\(^8\)

\[
(7) \quad x_t = a_t x_{t-1} + \varepsilon_t
\]

as a VQ(1) representation:

\[
(8) \quad X_t = \Phi_\varepsilon^{-1} \Phi_1 X_{t-1} + \Phi_\varepsilon^{-1} \varepsilon_t
\]

and then analyze whether the root of the corresponding characteristic equation

\[
(9) \quad \left| \Phi_\varepsilon - \Phi_1 Z \right| = 1 - \alpha_1 \alpha_2 \alpha_3 \alpha_4 = 0
\]

is outside the unit circle, that is, if \( g(\alpha) < 1 \), with \( g(\alpha) = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \). In other words, the following test is performed:

\[
(10) \quad H_0: g(\alpha) = \Pi_{\alpha_s} \alpha_s = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \\
H_1: \left| g(\alpha) \right| < 1 \text{ is periodically stationary}
\]

Once \( H_0 \) could not be rejected we have to investigate whether the hypotheses (a)-(b) are true:

\[
(11) \quad H_0: \alpha_s = 1 \quad s = 2,3,4 \quad (a) \\
H_0: \alpha_s = -1 \quad s = 2,3,4 \quad (b)
\]

given that \( \alpha_1 \alpha_2 \alpha_3 \alpha_4 = 1 \) implies either \( \alpha_4 = 1 \) or \( \alpha_4 = -1 \). The first one reduces the difference filter to \( (1-B) \) while the second one reduces the difference filter to \( (1+B) \), which is the corresponding filter to a seasonal root \( -1 \). When the null hypothesis in (a) could not be rejected, the PAR(1) process has a non seasonal unit root, that is to say, it is a PAR(1) process of a I(1) time series and is usually called PARI. When both null hypothesis (a) and (b) could be rejected, it is an AR(1) model periodically integrated, usually referred to as PIAR(1).

According to the results presented in Chart 4, REER, productivity, Government consumption (as a share of GDP), LIBOR, net interest rate and TOT can be described as autoregressive models periodically integrated, that is to say, as non stationary autoregressive time series that need a filter of the type \( (1-\alpha_s B) \) to remove their stochastic trend, being \( \alpha_s \) the autoregressive parameters. In the next session, it will be analyzed the possible existence of periodic cointegration among them. The orders of the periodic AR

\(^7\) For G, Schwarz Criterion is used, given that \( F_{\text{PAR}} \) is rejected.

\(^8\) For PAR(2) processes, the notation is similar. See Franses (1996).

\(^9\) Some of the values of \( \alpha \) may be greater than unity.
models do not necessarily have to be equal, nor do the lengths of the autoregressive polynomials in each of the seasons have to be equal.

---

**Chart 4 – PERIODIC UNIT ROOT TEST**

Two-step procedure

<table>
<thead>
<tr>
<th>Time Series¹</th>
<th>Sample value ²</th>
<th>Statistics³</th>
<th>Sample value</th>
<th>Critical Value ⁴</th>
<th>Sample value</th>
<th>Critical Value ⁵</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LRj</td>
<td>LRj</td>
<td>N(g(α)-1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REER</td>
<td>0.9429</td>
<td>0.06</td>
<td>-0.26</td>
<td>-0.95</td>
<td>2.37</td>
<td>2.70(5%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.98(1%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π</td>
<td>0.6549</td>
<td>5.02</td>
<td>-2.24</td>
<td>-35.55</td>
<td>3.65*</td>
<td>2.70(5%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.98(1%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>-0.0000</td>
<td>0.06</td>
<td>-0.25</td>
<td>-101.0</td>
<td>9.24¹</td>
<td>2.70(5%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.98(1%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i*</td>
<td>0.9789</td>
<td>0.31</td>
<td>-0.56</td>
<td>-2.51</td>
<td>2.38**</td>
<td>2.70(5%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.98(1%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in</td>
<td>0.9500</td>
<td>1.34</td>
<td>1.16</td>
<td>-4.75</td>
<td>4.89**</td>
<td>2.70(5%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.98(1%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOT⁶</td>
<td>-0.0010</td>
<td>4.00</td>
<td>-2.00</td>
<td>-118.1</td>
<td>63.64**</td>
<td>2.70(5%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.98(1%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:

(1) Original time series, without any previous transformation. REER, is real effective exchange rate, in logs; π is productivity, a difference of log variables; G, Government consumption as a ratio of GDP; i* is foreign interest rate (LIBOR); in is net interest rate; TOT is terms of trade, in logs.

(2) It is calculated as the product of estimated parameters shown in Chart 3, except for G and TOT.

(3) The statistics are:

3.1 **Likelihood ratio (LR):** \( LR_j = n \ln (SSR_0 / SSR_j) \), obtained by comparing SSR (sum of squared residuals) of a LS estimation on \( x_i = \Sigma \alpha \beta_i x_{i-1} + \varepsilon_i \), with respect to the SSR of a LS estimation imposing \( H_0: x_i = \Sigma \alpha \beta_i x_{i-1} + \varepsilon_i \). In both cases, intercepts and trend variables are added when needed.

3.2 \( LR_j = (\text{sgn}(g(\alpha))-1)LR_{1-B}^{1/2} \), with \( g(\alpha) \) evaluated at \( H_0 \). The 5 and 10 per cent asymptotic critical values are 9.24 and 7.52 (LRi) and –2.86 –2.57 (LRit).

3.3 \( N(g(\alpha)-1) \) is similar to the one used for non periodic AR models scaled by \( N \) in order to compare with values reported by Fuller (1976), charts 8.5.1 and 8.5.2.

(4) Statistics to the parameter restrictions related to the filter (1-B) are distributed F(3, n-k).

(5) Statistics to the parameter restrictions related to the filter (1+B) are distributed F(3, n-k).

(6) As TOT is a PAR(2) process it can be written in its periodically differenced form as:

\[
(TOT - TOT_{i-1}) = \mu - \beta_i (TOT - TOT_{i-1}) + \varepsilon_i,
\]

where \( a_i = a_i \) and \( \beta_i = \beta_i \) for \( kCN \) and \( s=1, 2, 3, 4 \). The investigation of the presence of unit roots amounts to investigate the solution to the characteristic equation, which is reduced to:

\[
(1 - \beta_i \beta_i \beta_i - \beta_i - \beta_i) = 0
\]

Hence, TOT has a UR either when \( \beta_i = 1 \) or \( \beta_i = 1 \) and at most two UR when both products equal unity. (See Franses (1996), pp 99). The same reasoning applies to G.

(7) When \( H_0: \alpha_s = 1 \) cannot be rejected, it is a PAR(ρ) process, that is, a PAR(ρ) process for an I(1) time series; when both \( H_0: \alpha_s = 1 \) and \( \alpha_s = 1 \) are rejected, it is an AR process of order \( \rho \) for a periodically integrated time series, PIAR(ρ).

* Significant at 5%
** Significant at 1%
IV. PERIODIC COINTEGRATION APPROACH: APPLICATION TO REER

In this session, I will try to find a stable representation of the REER with respect to its fundamentals (given by the literature) that takes into account the statistical characteristics of the time series that were depicted in the previous session. That is to say, I will try to find an equation that combines the variable seasonal patterns of the time series involved in a way that renders stable but changing long-run relationships and adjustment parameters that change with the season.

IV.1 Definitions
Definition. Two periodically integrated processes $y_t$ and $x_t$ are cointegrated when the linear combination $y_t - \theta_s x_t$ can be described using a periodically stationary process, where $\theta_s$ is a seasonally varying parameter. (Franses, 1996, pp. 181).

As Franses (1996) argues, the common trend property of two PIAR series implies that there is a single unity solution to the characteristic equation for a VQ model for vector processes $Z_t$. And a natural and simple test procedure seems to be given by evaluating the residuals from the OLS regression

$$y_t = \hat{\mu}_t + \hat{\theta}_s x_t + \hat{u}_t$$

(12)

Franses (1996) suggests a two-step procedure similar to the Engle-Granger approach for the non-periodic analysis which is the one that will be performed in this paper.

IV.2 The estimation

In the first step of the estimation procedure followed here, four long-run relationships are found, one for each quarter (season) $s$:

$$\text{REER}_s = \theta_{s0} + \theta_{s1} \text{TOT}_s + \theta_{s2} \text{in}_s + \theta_{s3} \text{G}_s + \varepsilon_s$$

(13)

Next, a periodic error correction equation is presented:

$$\Delta \text{REER}_t = \sum_{s=1}^{4} D_{s} \lambda_s \left( \text{REER}_{t-s} - \theta_{s0} - \theta_{s1} \text{TOT}_{t-s} - \theta_{s2} \text{in}_{t-s} - \theta_{s3} \text{G}_{t-s} \right) + \sum_{s} \Delta \text{REER}_{s,t} +$$

$$+ \sum_{s} \beta_s \Delta \text{TOT}_{s,t} + \sum_{s} \Delta \text{in}_{s,t} + \sum_{s} \Delta \text{G}_{s,t} + \sum_{s} \Delta X_{s,t} + \varepsilon_t$$

where X is a vector of exogenous variables. Then, the existence of total or partial periodic cointegration is tested. The variables that stand before each long-run relationship, $\lambda_s$, are strictly negative as needed in order to verify cointegrating relationships.

---

10 Franses (1996) distinguishes between periodic cointegration models (PCM) and periodic error-correction models (PECM): a PCM has seasonally varying cointegration relationships while in a PECM only the adjustment parameters are seasonally variable.

11 There could be contemporaneous variables, provided they are weakly exogenous.
**Chart 5- PERIODIC COINTEGRATION ANALYSIS**  
**Dependent variable: ∆₄ REER**  
**Sample: 1983:1-2006:4**

### Partial cointegration test

<table>
<thead>
<tr>
<th>Season</th>
<th>(W_s)</th>
<th>Critical value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.46</td>
<td>8.10</td>
<td>(H_0) of no periodic cointegration in Q1 is rejected at 5%</td>
</tr>
<tr>
<td>2</td>
<td>9.79</td>
<td>8.10</td>
<td>(H_0) of no periodic cointegration in Q2 is rejected at 5%</td>
</tr>
<tr>
<td>3</td>
<td>7.85</td>
<td>6.48</td>
<td>(H_0) of no periodic cointegration in Q3 is rejected at 10%</td>
</tr>
<tr>
<td>4</td>
<td>33.91</td>
<td>8.10</td>
<td>(H_0) of no periodic cointegration in Q4 is rejected at 5%</td>
</tr>
</tbody>
</table>

### Total cointegration test

<table>
<thead>
<tr>
<th>(W^2)</th>
<th>Critical value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>761.14</td>
<td>21.65</td>
<td>(H_0) of no periodic cointegration in all quarters is rejected at 5%</td>
</tr>
</tbody>
</table>

**Notes:**

1. Wald test for partial cointegration for season \(s\):
   \[
   W_s = (n - l) \left( \frac{SSRO_s - SSR_1}{SSR_1} \right)
   \]
   where \(n\) is the number of observations, \(l\) is the number of estimated parameters under \(H_1\), \(SSRO_s\) and \(SSR_1\) are the sum of squared residuals from the LS regressions under \(H_0\) and \(H_1\) respectively, for \(s = 1, 2, 3, 4\).
2. Wald test for total cointegration:
   \[
   W = (n - l) \left( \frac{SSR_0 - SSR_1}{SSR_1} \right)
   \]
   where \(n\) is the number of observations, \(l\) is the number of estimated parameters under \(H_1\), \(SSR_0\) and \(SSR_1\) are the sum of squared residuals from the LS regressions under \(H_0\) and \(H_1\) respectively.
3. Asymptotic distributions for \(W_s\) and \(W\) statistics are not standard and they were derived by Boswijk and Franses (1995). I report the corresponding critical values for two (one weakly exogenous) variables and a significance level of 5% and 10%.

The empirical evidence for Uruguay in the 1983:1-2006:4 period allows us to reject the absence of partial and total periodic cointegration hypothesis for the real effective exchange rate and its fundamentals. Besides, it is not possible to reject some restrictions on the adjustment parameters and long-run elasticities as well (See Chart 6). Finally, we arrive at the following PCM specification (standard deviations between parenthesis)\(^{12}\):

\(^{12}\) Dummy variables were added to the long-run relationships in: 1990:1 and 1990:3 (regional crisis) and 2002.2 (abandonment of nominal exchange rate target zone in Uruguay) and 2003.1 (domestic crisis that followed). Also, in order to improve short-run adjustment, outliers were taken care of when the corresponding error was greater than three times the standard error of the regression. That happened in: 1985:4, 1994:4, 2002:3, 2003:3, 2003:4 and 2004:3.
\( \Delta_t \text{REER} = -0.77 (\text{REER} - 6.45 + 0.20 \text{TOT} + 0.01 \text{in} + 0.98 G)_{t-1} + 0.45 (\text{REER} - 6.34 + 0.37 \text{TOT} + 0.009 \text{in})_{t-4} + \)
\[
-0.45 (\text{REER} - 7.18 + 0.20 \text{TOT} + 0.005 \text{in} + 1.80 G)_{t-4} - 0.77 (\text{REER} - 5.78 + 0.25 \text{TOT} + 0.01 \text{in})_{t-4} + \\
+ 0.71 \Delta_t \text{REER}, -0.004 (\Delta_t E_{t-3} - \Delta_t E_{t-4}) + 0.69 (\Delta_t p^*_t - \Delta_t p_{t-1}) + 0.27 (\Delta_t p_{t-1} - \Delta_t p_{t-2}) + \\
-0.15 (\Delta_t \text{TOT}_{t-1} - \Delta_t \text{TOT}_{t-4}) + \epsilon_t
\]

where \( E \) is nominal exchange rate (Uruguayan pesos per US dollars), \( p^* \) is foreign price aggregate index and \( p \) is (domestic) Consumer Price Index. The corresponding diagnostics are:

\[
\overline{R}^2 = 0.93; \quad \sigma = 3.1\%; \quad F_{\text{JB},1-1} = 0.00; \quad F_{\text{JB},1-4} = 0.00; \\
F_{\text{ARCH},1-4} = 3.37; \quad F_{\text{ARCH},1-4} = 2.25; \quad JB = 4.23
\]

which indicate a relatively good adjustment with a rather high standard deviation, though. Residuals are stationary, homoskedastic and normally distributed. As the only contemporaneous variable with the REER is TOT, it is tested weak exogeneity of TOT with respect to the parameters of the long-run relationship. To do so, an AR(4) model for \( \Delta_t \text{TOT} \) is regressed adding the cointegrating vectors from the PCM just found. The F statistic is lower than table values at 1% so it could not be rejected the hypothesis of weak exogeneity of TOT to long-run parameters determination. That is why the single-equation approach used in this study seems to be appropriate. Also, Granger non-causality from \( \text{TOT}, \text{in} \) and \( G \) to \( \text{REER} \) was rejected in the long-run relationships. As a result, this specification allows the investigator to make inferences and make forecasts as well.
In Chart 6 the results of restriction tests on parameters are presented. It can be seen that: (a) adjustment coefficients reflect variable cost adjustments according to the quarter, but are the same for the first and fourth quarter and the second and third one; (b) long-run elasticities are not the same, except for \( TOT \) in Q1 and Q3; (c) \( TOT \) and \( in \) are always real fundamentals for the REER while \( G \) appears to have a stable long-run relationship with REER only in the first and third quarters, being greater in the latter one. This means that the objective relationships are changing according to the quarter.

<table>
<thead>
<tr>
<th>Chart 6 – PARAMETER RESTRICTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong> ( \Delta 4 \text{ REER} )</td>
</tr>
<tr>
<td><strong>Sample:</strong> 1983:1-2006:4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Restriction</th>
<th>Sample value(^1)</th>
<th>Critical values(^2)</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0: \lambda_1 = \lambda_4 )</td>
<td>0.56</td>
<td>4.03 7.19</td>
<td>The ( H_0 ) of equal adjustment coefficients at Q1 and Q4 could not be rejected at 1%</td>
</tr>
<tr>
<td>( H_0: \lambda_2 = \lambda_3 )</td>
<td>0.04</td>
<td>4.03 7.19</td>
<td>The ( H_0 ) of equal adjustment coefficients at Q2 and Q3 could not be rejected at 1%</td>
</tr>
<tr>
<td>( H_0: \lambda_1 = \lambda_4 ), ( \lambda_2 = \lambda_3 )</td>
<td>0.31</td>
<td>3.18 5.06</td>
<td>The ( H_0 ) of equal adjustment coefficients at Q1 and Q4 and of Q2 and Q3, could not be rejected at 1%</td>
</tr>
<tr>
<td>( H_0: \theta_1 = \theta_3 )</td>
<td>3.30</td>
<td>4.03 7.19</td>
<td>The ( H_0 ) of equal long-run elasticity of ( TOT ) to REER at Q1 and Q3 could not be rejected at 1%</td>
</tr>
<tr>
<td>( H_0: \lambda_1 = \lambda_4 ), ( \lambda_2 = \lambda_3 ), ( \theta_1 = \theta_3 )</td>
<td>1.61</td>
<td>2.79 4.20</td>
<td>The ( H_0 ) of equal adjustment coefficients at Q1 and Q4 and of Q2 and Q3 and that long-run elasticities of ( TOT ) to REER at Q1 and Q3 are equal, could not be rejected at 1%</td>
</tr>
</tbody>
</table>

**Notes:**
1. It corresponds to a standard “F-test” on the periodically cointegrated equation.
2. At 5% and 1%, respectively.

It seems as if it is relatively simple to adequately describe the REER for Uruguay through a periodic cointegration approach. The final periodic cointegration model has four long-run target relationships:

\[
(15.1) \quad \text{REER} = 630 \quad TOT^{-0.20} e^{-0.01in} e^{-0.08G} \quad \text{in the first quarter}
\]
\[
(15.2) \quad \text{REER} = 565 \quad TOT^{-0.37} e^{-0.008in} \quad \text{in the second quarter}
\]
\[
(15.3) \quad \text{REER} = 1307 \quad TOT^{-0.20} e^{-0.005in} e^{-1.80G} \quad \text{in the third quarter}
\]
\[
(15.4) \quad \text{REER} = 322 \quad TOT^{-0.25} e^{-0.01in} \quad \text{in the fourth quarter}
\]

It means that, ceteris paribus, a change in terms of trade has its major impact in the third quarter despite the fact that the higher long-run elasticity occurs in the second one.

The speed of adjustment of misalignments is different depending on the quarter. The effects of any shock that occurs in Q1 and Q4 dissipate faster than if it took place in Q2 and Q3. In fact, it could take either four quarters to return to equilibrium (Q1 an Q4) or ten ones (Q2 and Q3). It seems as if October-March period (Q4 and Q1) is the “high-speed time” for disequilibrium adjustments.
V. CONCLUDING REMARKS

In this paper, REER for Uruguay and its long-run fundamentals have been viewed as periodically integrated time series that could combine their changing seasonal patterns in such a way that stable but quarterly-changing objective relationships could be achieved and adjustment parameters that change with the season (quarter) could be found. This approach, by recognizing the stochastic nature of the seasonal pattern of the time series involved, avoids inconsistent estimations, errors in statistical inference and also biases in economic policy decisions.

The periodic cointegration model finally estimated shows that REER in Uruguay for 1983:1-2006:4 can be treated as a stationary time series in fourth differences, that is to say, shocks that affect the annual change of REER have only transitory effects and in the long run REER would achieve the value given by its fundamentals. Besides, it seems as if REER has different fundamentals, depending on the quarter (season). In addition, the impact of changes in those fundamentals on the long-run target relationships depends on which quarter those changes take place. And last but not least, the adjustment of equilibrium misalignments are faster if they take place in the October-March period than in the rest of the year. In effect, a 77-percent disequilibrium dissipation would take either one quarter (Q1 and Q4) or two quarters and a half (Q2 and Q3); a 90-percent disequilibrium dissipation would take either three and ½ quarters (Q1 and Q4) or eight quarters (Q2 and Q3) and a total disequilibrium dissipation would take either four quarters (Q1 and Q4) or ten quarters (Q2 and Q3).

Real exchange rate recent history plays a very important role in short-run dynamics as well as annual inflation both domestic and foreign. Terms of trade is the only long-run fundamental that is also present in the determination of REER in the short-run.

Some outcomes of this investigation are useful for economic policy. There are just a few variables that are significant in explaining the evolution of REER and that the policy maker can control. They are $G$, $E$ and $\pi$, that is, Government consumption as a share of GDP, nominal exchange rate (depending on the exchange rate policy) and domestic inflation (to a certain extent$^{13}$). It seems that reductions in Government size could increase REER in the long run as well as reductions in net interest rate$^{14}$; changes in nominal exchange rate or in domestic inflation could only affect short-run dynamics but they could not influence fundamental determinants of REER, which seems to be a stationary process.

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$^{13}$ The inflationary process depends on variables that are not under the direct control of the policy maker.

$^{14}$ Induced by reductions in domestic interest rate.
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