Abstract

This paper examines how aggregate political budget cycles (aggregate PBC) are affected by checks and balances when the political parties are office motivated. When the legislature has to authorize new debt, there are no PBC if there is divided government and perfect compliance with the budget law. PBC are only possible when there is unified government or low compliance with the budgetary law. What drives these results are effective checks and balances, that provide a commitment device to solve the credibility problem behind PBC. An extension of the basic model is to analyze the tradeoff voters face when divided government comes at the cost of government efficiency.

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1 Introduction

The standard results on rational PBC not only require asymmetric information, but also a fiscal authority with discretion over fiscal policy. Once one drops the assumption of a single fiscal authority, the possibility of PBC will depend on the leeway that the legislature allows the executive in pursuing electoral destabilization (Streb 2005).

Saporiti and Streb (2004) formally analyze the implications for PBC of considering that in constitutional democracies the process of drafting, revising, approving and implementing the budget requires the concourse of the legislature. Their approach relies on the Romer and Rosenthal (1978) and (1979) agenda setter model; Persson, Roland and Tabellini (1997) use a similar framework to analyze how to control the rents of politicians through separation of powers. The executive wants to win elections, and the legislature represents the interests of the people. In a setup with asymmetric information on the budgetary process similar to the Lohmann (1998a) timing, separation of powers is needed to make the budget rule credible, i.e., to commit the executive to not distort the composition of government spending.
towards visible items in electoral periods. Though they show the legislature has a moderating role on electoral cycles, compliance of the executive with the budget law is required. Otherwise, the process is moot.

The approach developed in this paper emphasizes electoral competition between political parties. Separation of powers has a bite in the fiscal process when the executive and legislative branches are not perfectly aligned. This draws on the insight of Alesina and Rosenthal (1995) on the moderating influence of an opposition legislature. Through the metric of veto players (Tsebelis 2002), this insight applies not only to divided government in presidential systems, but more generally to coalition governments. Coalition members start to compete among themselves for votes, so it is particularly hard for different political parties to collude close to elections.

We now sketch the relationship between separation of powers and PBC that we model here. Suppose, as in Shi and Svensson (2006) and Alt and Lassen (2005), that deficit financing is distortionary. However, there is asymmetric information and the competency of the executive just before elections matters for performance after elections. In electoral periods, the executive will be tempted to increase expenditure and reduce taxes in electoral periods to increase its electoral chances. Hence, the ex-ante optimal policy is not credible ex-post.

Let the legislature have the veto power to reject new indebtedness, as is standard in all budget rules. In a non-electoral period, the executive has no incentive to distort the optimal budget, because whatever it does that period will not affect its future electoral chances. Hence, optimal policy will be implemented in non-electoral periods. In an electoral period, the executive incumbent will still be tempted to distort the budget. If the legislature is aligned with the executive, it will not curb cycles because it shares its same electoral objectives. However, if the legislature is not aligned with the executive, it will not be interested in increasing the chances of success of the executive, so it will veto electoral changes in the budget. For this veto power to be effective in avoiding PBC, the legislature needs the oversight and enforcement capacity to insure that the executive complies with the approved budget law.

Following this interpretation, one can derive a sharp empirical implication for aggregate PBC: if there is perfect compliance with the budget law, the budget rule is credible if the party of the executive’s leader does not control the legislature. On the other hand, if there is imperfect compliance, the budget rule is never credible. Consequently, PBC should be larger either in countries with low legislative checks and balances, or with low observance of the rule of law.
2 Checks and balances

Consider an infinite-horizon society composed by a large but finite number of identical individuals, labeled \( i = 1, 2, \ldots, n \). Let \( t \) denote time, where odd positive integers are electoral periods and even positive integers are non-electoral periods.

2.1 Preferences

In every period \( t \), individual \( i \) plays roles both as a consumer and as a citizen. The representative consumer derives utility from a public good \( g_t \) and a private good \( c_t \). The representative consumer’s per-period payoff is given by a quasi-linear utility function,

\[
u(c_t, g_t) = c_t + \alpha \ln(g_t),\]

where \( 0 < \alpha < 1 \). The intertemporal utility function \( U \) is given by

\[
U = \sum_{t=0}^{\infty} \beta^t u(c_t, g_t), \quad 0 < \beta < 1.
\]

Output \( y_t \) is exogenous, with \( y_t = y \). By the consumer’s per-period budget constraint, consumption \( c_t \) equals disposable income, namely, \( y \) net of government impositions \( p_t \):

\[
c_t = y - p_t.
\]

2.2 Government budget

Each period \( t \), the government is subject to the budget constraint

\[
\gamma_t = \pi_t + d_t - (1 + r)d_{t-1},
\]

where \( \gamma_t \) denotes actual budget expenditures on public goods, \( \pi_t \) are tax revenues, \( d_t \) is public debt and \( r \) is the interest rate on debt, that is constant\(^1\).

Public resources \( \gamma_t \) are transformed into the public good \( g_t \) according to the competence \( \theta_t \) of the government:

\[
g_t = \theta_t \gamma_t.
\]

Similarly, the competence of the government affects how impositions \( p_t \) become actual government tax receipts \( \pi_t \), reflecting, among other things, the use of more or less distortionary taxes:

\[
p_t = \frac{\pi_t}{\theta_t}.
\]

\(^1\)We consider that government could be a net lender, i.e. \( d_t < 0 \), in which case we assume that the relevant interest rate, not necessarily equal to \( r \), is \( r' \).
By (5), to provide a given level of public goods expenditure must be higher with less competent governments. Similarly by (6), with a given level of impositions less competent governments generate less tax receipts.

The representative individual cares about the competence of the incumbent in providing public goods. Since the incumbent does not know its competence when it takes budget decisions, from its viewpoint the electoral outcome is uncertain.

Since voters are inclined to reelect more competent incumbents, this creates an electoral incentive for governments to lower taxes in electoral years. It also creates an incentive to increase expenditure using debt finance.

In contrast, in Shi and Svensson (2002), and Alt and Lassen (2006), electoral cycles in the budget balance are exclusively through expenditure cycles, not tax and expenditure cycles. Here, taxes and expenditures fluctuate with the competence of the government.

### 2.3 Government competence

We assume that the competence of the government depends on the competence of the party that controls the executive branch $E$. For each party $i = A, B$, the competence shock $\varepsilon^i_t$ is a random i.i.d. variable which is uniformly distributed over the interval $\left[-\frac{1}{2\xi}, \frac{1}{2\xi}\right]$, with expected value $E(\varepsilon) = 0$ and density function $\xi > 0$. A higher value of $\varepsilon^i_t$ corresponds to a more competent politician. The probability distribution of competence $\theta^i_t$ conditional on $\varepsilon^i_{t-1}$, $F(\theta^i_t \mid \varepsilon^i_{t-1})$, is also uniform, with support $\left[\bar{\theta} + \varepsilon^i_{t-1} - \frac{1}{2\xi}, \bar{\theta} + \varepsilon^i_{t-1} + \frac{1}{2\xi}\right]$, and $E(\theta^i_t \mid \varepsilon^i_{t-1}) = \bar{\theta} + \varepsilon^i_{t-1}$.

Actual competence of the government is partially lasting, following a first-order moving average process as in Rogoff and Sibert (1988) and others:

\[
\theta_t = \bar{\theta} + \varepsilon_{t-1} + \varepsilon_t. \tag{7}
\]

Henceforth, $\bar{\theta} > 1/\xi$, so $\theta_t > 0$ and (5) and (6) are well-defined.

### 2.4 Discount factor and interest rates

Following the insight in Shi and Svensson (2006), the quasilinear preferences in (1), jointly with an assumption about the value of the discount factor $\beta$
and the interest rates, can drastically simplify the optimal policy problem. Whereas Shi and Svensson (2006) assume the interest rate is increasing in the level of debt, we assume that the rate \( r > 0 \) at which the government can borrow is constant, but this borrowing rate is larger than the rate \( r' > 0 \) at which it can lend.

Furthermore, we assume the following condition is satisfied:

\[
\frac{1}{1 + r} E_t \left( \frac{1}{\bar{\theta}_t} \right) < \beta < \frac{1}{1 + r'} E_t \left( \frac{1}{\bar{\theta}_{t+1}} \right). \tag{8}
\]

This condition will assure that neither debt nor holding financial assets will be optimal in equilibrium.

Also, let

\[
(1 + r) > (1 + r') \left( \frac{\bar{\theta} + \frac{1}{\xi}}{\bar{\theta} - \frac{1}{\xi}} \right)^2. \tag{9}
\]

Given this, since \( \frac{\bar{\theta} - \frac{1}{\xi}}{\bar{\theta} + \frac{1}{\xi}} < \frac{E_t \left( \frac{\bar{\theta}}{\bar{\theta}_{t+1}} \right)}{E_t \left( \frac{1}{\bar{\theta}_{t+1}} \right)} < \frac{\bar{\theta} + \frac{1}{\xi}}{\bar{\theta} - \frac{1}{\xi}} \), a sufficient condition for (8) to hold is

\[
\frac{1}{1 + r} \frac{\bar{\theta} + \frac{1}{\xi}}{\bar{\theta} - \frac{1}{\xi}} < \beta < \frac{1}{1 + r'} \frac{\bar{\theta} - \frac{1}{\xi}}{\bar{\theta} + \frac{1}{\xi}}. \tag{10}
\]

Given the parameter values of the interest rates in (9) and the discount factor in (10), condition (8) will be met.

### 2.5 Asymmetric Information

The timing, as in Lohmann (1998b), is that in each period \( t \), incumbents do not observe the value of \( \varepsilon_t \) before making budget decisions. The interpretation of this timing is that policy is decided under uncertainty, so it leads to a lottery of outcomes. If decisions were taken instead under certainty, the choice of the policy instrument would be the choice of the outcome.

The representative (median) voter does not observe either the executive leader’s most recent competence shock, \( \varepsilon_t \), or the budget decisions \( (\gamma_t, \pi_t, d_t) \) before voting. The only information it receives is the amount of public good \( g_t \) that is provided, and of tax payments \( p_t \) it makes. Thus, incumbents have a temporary information advantage over the actual budget allocation implemented. All past competence shocks are common knowledge. We assume voters know the incentives political parties face and the objectives they try to achieve.\(^4\)

\(^4\)It is assumed that the median voter knows the parameters of the budget process.
2.6 Veto players

The agenda setter model of Romer and Rosenthal (1978, 1979) allows to reduce the policy-making process carried out to set the budget to the interaction of the current leaders of the two branches of government, the executive and the legislature.

The terms in office in the executive and legislative branches last two periods (we are abstracting from midterm legislative elections). Every other period, the electorate removes or confirms the executive and legislative leaders in an explicit electoral contest. If the incumbent is confirmed, it controls this branch for another term. Otherwise, the opposition takes office.

We assume there are two parties, A and B. A party’s payoffs are as follows. Besides caring about the utility from the consumption of private and public goods, when a party wins executive elections and its leader becomes the E incumbent, it receives an exogenous rent $\chi^E > 0$ at the beginning of each term in office. The party that wins legislative elections and controls $L$ receives a rent $\chi^L \geq 0$, where $\chi^L < \chi^E$. These rents reflect the strength of the electoral goal, to use Lohmann’s (1998b) words, and will be the source of conflict between political parties and the electorate.

Through the idea of veto players, the agenda setter model can be used to reflect not only presidential systems, but also the working of parliamentary systems (Tsebelis 2002). While in a presidential system, $E$ is the leader of the executive and $L$ is the leader of the legislature, in a parliamentary system $E$ can be taken to represent the leader of the majority coalition party, and $L$ the leader of the minority coalition party.

If $E$ and $L$ are controlled by the same party, there is no veto player: in a presidential system, this is referred to as unified government, when the executive has an aligned legislature; in a parliamentary system, as single-party rule where one party has a majority of seats in the legislature. There are veto players in a presidential system when there is divided government, and the legislature is controlled by opposition party whose electoral motives are strictly opposed to those of the executive; in a parliamentary system, something similar happens when the party that leads government is forced to form a coalition to reach a majority of seats in parliament.

2.7 Budget process

The process for setting the budget involves a specific system of checks and balances. At the stage of budget formulation and approval, $E$ makes a budget allocation proposal, which must be accepted by $L$ to become law. We first assume no amendment rights exist, so $L$ faces a take-it-or-leave-it proposal where the reversion outcome (the status quo) in case of rejection is specified below. The proposals are in terms of budget expenditure and
debt, because the budget restriction determines the required tax revenues (only two of these three variables can be chosen freely).

- The timing of the budget process in period \( t \) is as follows:

1. \( E \) proposes \( \hat{\gamma}_t^E, \hat{d}_t^E \) to \( L \).

2. Since \( L \) has no amendment rights, \( L \) chooses whether to accept proposal or not. If the proposal is not accepted, the budget is given by status quo \( \bar{\gamma}_t, \bar{d}_t \). This will determine the approved budget \( \hat{\gamma}_t, \hat{d}_t \).

3. \( E \) implements \( \gamma_t, d_t \), which equals the approved budget under perfect compliance (below we will consider the case of imperfect compliance).

4. \( \varepsilon_t \) is realized and \( g_t \) and \( p_t \) are determined according to (5) and (6); 

5. Voters observes \( g_t \) and \( p_t \), but not \( \varepsilon_t \) nor \( (\gamma_t, \pi_t, d_t) \), forming a belief \( \hat{\theta}_t \) about the incumbent’s competency.

6. Without loss of generality, we assume party \( A \) controls \( E \). If \( t \) is an odd positive integer, i.e., an electoral period, voters decide whether to reelect party \( A \) in \( E \), and whether to vote incumbent party \( A \) or opposition party \( B \) for \( L \).

7. Individuals observe \( \varepsilon_t \) and \( (\gamma_t, \pi_t, d_t) \) and period \( t \) ends.

### 2.8 Budget rule

As is standard in the agenda setting model, if the executive’s budget proposal is rejected, the status quo for expenditure is given by an exogenous reversion outcome:

\[
\bar{\gamma}_t = \hat{\gamma}_t. \tag{11}
\]

As to the status quo for debt, we assume that there is an endogenous debt ceiling:

\[
\bar{d}_t \leq d_{t-1}. \tag{12}
\]

This endogenous debt ceiling merely reflects the restriction that, unless authorized by \( L \), there can be no new debt. This budget rule is typical of all budget processes.

Though taxes, just like new debt, must be authorized by the legislature, for simplicity the budget rules in the model do not restrain the executive in relation to the amount of tax receipts. Since in equilibrium the executive will have no incentive to tax beyond expenditure (or, if there is outstanding debt, beyond expenditure plus the amount needed to rescue outstanding debt), it is not necessary to explicitly introduce this restriction.
3 Equilibrium

We first study the case when there are no elections, as a benchmark.

3.1 No elections

A candidate is randomly selected in period $t = 0$, and he remains in office forever. By quasilinear preferences, the marginal utility of consumption is equal to one. If, in expected value, the marginal utility of the government consumption good is equal to the marginal utility of consumption, any extra resources the government may have will be optimally used to reduce taxes.

Suppose the government resorts to an extra dollar of debt in period $t$ to reduce taxes. From expressions (1), (2), (3) and (6), it follows that expected utility increases $E_t \left( \frac{1}{\theta_t} \right)$ in period $t$. If the extra dollar of debt is repaid next period, utility falls by $(1 + r)E_t \left( \frac{1}{\theta_{t+1}} \right)$ in period $t+1$. Since the future is discounted at the rate $\beta$, by (8) it will never be optimal to borrow an extra dollar and repay it in the next period because:

$$\beta(1 + r)E_t \left( \frac{1}{\theta_{t+1}} \right) > E_t \left( \frac{1}{\theta_t} \right)$$

Here $E_t \left( \frac{1}{\theta_{t+1}} \right)$ equals unconditional expectation, since there is no information on current shock when decision is taken, so expectation for $t+1$ equals expectation for all future time periods. This condition also rules out that repaying the debt farther out in the future is optimal, because $(1 + r) > 1$, so the compounding effect makes the condition more binding for $t > t + 2$. Following an analogous argument, condition (8) also rules out the possibility that the government may become a net lender. This leads to a corner solution with no debt nor financial assets.

Since our assumptions about $\beta$, $r$ and $r'$ in (8) assure that $d_t = 0$ (i.e., $\gamma_t = \pi_t$) for $t = 0, 1, ...$, the intertemporal problem can be broken in a sequence of simpler optimization problems.

$$\max_{\{\gamma_t, \pi_t\}} E_t[c_t + \alpha \ln(g_t)]$$

s.t. (3), (4), (5) and (6).

The solution, using the properties of the uniform distribution, and then integrating, is:

**Proposition 1** Without elections, the executive will choose optimal expenditure and tax collection each period:

$$\gamma_t^* = \pi_t^* = \frac{\alpha}{E_t[\frac{1}{\theta_{t+\epsilon_t} + \epsilon_t - 1}]} = \frac{\alpha}{\xi \ln \left( \frac{\theta_{t+\epsilon_t} + \epsilon_t}{\theta_{t+\epsilon_t} - \frac{1}{\xi}} \right)}, \quad t = 0, 1, \ldots$$

(13)
Fiscal policy $\gamma_t^*$ and $\pi_t^*$ depend on expected competence, and are increasing in the past shock. Since the budget is decided ex-ante, it cannot be conditioned on actual competency. However, ex-post, a more competent incumbent generates a greater provision of the public good $g$ with a given $\gamma$ and imposes a lower burden on the tax payers, lowering the relative cost of public vs. private goods.

3.2 Unchecked executive

Consider next the model with regular elections every two periods. There is only one policy-maker, the executive. In period $t = 0$ a candidate is randomly selected for the executive. Subsequently, odd integers are electoral periods and even integers are non-electoral years.

The players are the incumbent party, the opposition party, the representative voter $V$, and Nature. Because the two parties only differ in competence, and these competence shocks are transitory, the solution of the infinite-horizon model described can be broken down into a sequence of steps. The solution can be found by backward induction, in four steps.

**Step 1: The incumbent’s decision in a nonelectoral period**

In period $t + 1$, a nonelectoral period, the incumbent (either $A$ or $B$) has no incentive to manipulate the voters’ perception of its competence, since the outcome of future elections will depend on the expected competence in $t + 3$, which is uncorrelated with competence in $t + 1$. Since the optimal strategies of all incumbents in the post-electoral period are the same, the distinction between the original and the potential incumbents is omitted to simplify the notation. Hence:

\[
\gamma_{ue}^{t+1} = \gamma^*_{t+1} = \frac{\alpha}{E_{t+1}[\pi_{t+1}]} = \frac{\alpha}{\xi \ln \left( \frac{\theta + \epsilon_{t+1}}{\theta + \epsilon_t} \right)} ,
\]

\[
\pi_{ue}^{t+1} = \gamma^*_{t+1} + (1 + r)d_t ,
\]

where the superscript $ue$ refers to unchecked executive.

Notice that in a nonelectoral period the expenditure is the same as in a setup without elections, but there may be more taxes if the incumbent has to pay off the debt incurred in the last election period.

**Step 2: The government’s plausibility restriction and the voter’s perception of government competence**

At election time, voters observe $g_t$, $\pi_t$, but not $d_t$, $\gamma_t$ and $\pi_t$. They know that consumers and the government are subject to restrictions (3), (4), (5), (6).

There is an additional restriction, that we label a “plausibility” restriction. If the incumbent relies on debt, it must preserve the ratio between expenditures on public goods and tax collection that would obtain without
electoral manipulation of the budget, to avoid making that action transparent to voters. This restricts the way the incumbent can use debt, forcing it to split it in specific proportions between more expenditure and less taxes.

Let \( \pi^*_t = \gamma^*_t \) denote the budget that is not affected by opportunistic concerns, the optimal budget choice when there is no previous debt. To derive the observed levels of \( g_t \) and \( p_t \) that satisfy our plausibility restriction, let the budget choices \( \gamma_t \) and \( \pi_t \) be given by a scale factor \( \omega_t \) that determines values of \( g_t \) and \( p_t \) possible under technological restrictions (5) and (6):

\[
\gamma_t = \omega_t \gamma^*_t, \quad \pi_t = \frac{\pi^*_t}{\omega_t}.
\]

This plausibility restriction implies that

\[
\frac{\gamma_t}{\pi_t} = \omega_t^2.
\]

This pattern of budget choices allows to replicate the original distribution of shocks, with the expected value of the distribution shifted to the right given \( \omega_t > 1 \) (the government is tempted to mimic positive competence shocks, not negative ones). That is to say, the budget items have to satisfy a certain ratio to replicate the distribution without electoral manipulation. This restriction implies that, beyond identity (4), debt must also satisfy:

\[
d_t = \gamma_t - \pi_t = (\omega_t^2 - 1) \pi_t = \left( \omega_t - \frac{1}{\omega_t} \right) \pi^*_t.
\]

This restriction makes it clear that debt must be used in predefined proportions to reduce taxes and increase expenditures, to preserve the characteristics of the original distribution of competency shocks.

Voters know this restriction on government actions and they include it in their estimation of the incumbent’s competence. If voters could observe \( \omega_t \), they could easy calculate \( \theta_t \), since \( \frac{\theta_t}{\pi_t} = \frac{\theta^*_t}{\pi^*_t} = \omega_t \), which implies that:

\[
\theta_t = \frac{\sqrt{\frac{\pi_t}{\pi^*_t}}}{\omega_t}.
\]

However, voters do not really observe \( \omega_t \). They must estimate it, in order to estimate \( \theta_t \). Therefore, if we call \( \hat{\omega}_t \) voters’ estimate of \( \omega_t \), the estimate \( \hat{\theta}_t \) of \( \theta_t \) is:

\[
\hat{\theta}_t = \frac{\sqrt{\frac{\pi_t}{\pi^*_t}}}{\hat{\omega}_t}.
\]

Using this expression, voters can estimate the incumbent’s current competence shock (\( \varepsilon_{t-1} \) is known in period \( t \)):

\[
\hat{\varepsilon}_t = \hat{\theta}_t - \bar{\theta} - \varepsilon_{t-1} = \frac{\sqrt{\frac{\pi_t}{\pi^*_t}}}{\hat{\omega}_t} - \bar{\theta} - \varepsilon_{t-1}.
\]
Step 3: The citizen’s vote

Voters compare the expected utility next period if they vote either the incumbent or the challenger. Voters can estimate the competence shock of the incumbent, but nothing can be concluded about the opposition from the observed policy actions of the government. In regard to the opposition, voters only know the distribution of $\varepsilon_t$ and hence that $E_t[\varepsilon_t] = 0$.

Expected utility from a vote for the incumbent is:

$$E_t[c_{t+1} + \alpha \ln(g_{t+1}) | \hat{\varepsilon}_t] = E_t[y - \frac{\gamma_{t+1}^{ue}}{\theta_{t+1}} + \alpha \ln(\theta_{t+1} \gamma_{t+1}^{ue}) | \hat{\varepsilon}_t]$$

(18)

Expected utility from a vote for the opposition is:

$$E_t[c_{t+1} + \alpha \ln(g_{t+1})] = E_t[y - \frac{\gamma_{t+1}^{ue}}{\theta_{t+1}} + \alpha \ln(\theta_{t+1} \gamma_{t+1}^{ue})]$$

(19)

In order to determine voters’ decision we must compare these two expressions. We formally do this comparison in the following proposition.

Proposition 2 $E_t[c_{t+1} + \alpha \ln(g_{t+1}) | \hat{\varepsilon}_t] \geq E_t[c_{t+1} + \alpha \ln(g_{t+1})]$ if and only if $\hat{\varepsilon}_t \geq 0$.

Corollary 1 Voters vote for the incumbent if and only if $\hat{\varepsilon}_t \geq 0$.

Proof 1 For a proof of the proposition please see appendix A. The proof of the corollary is immediate from (18) and (19).

We now employ this proposition to compute the probability that the incumbent wins the election. Let’s call this probability $\mu_t = \Pr(\hat{\varepsilon}_t > 0) = \Pr \left( \sqrt{\frac{g_t}{p_t}} \bar{\omega}_t - \tilde{\theta} - \varepsilon_{t-1} > 0 \right)$. Considering that the actual value of $\varepsilon_t$ equals $\sqrt{\frac{g_t}{p_t}} \bar{\omega}_t - \tilde{\theta} - \varepsilon_{t-1}$, adding this to both sides and simplifying, we get

$$\mu_t = \Pr \left[ \varepsilon_t > \sqrt{\frac{g_t}{p_t}} \left( \frac{1}{\omega_t} - \frac{1}{\bar{\omega}_t} \right) \right]$$.

Finally, considering that $\varepsilon_t$ follows a uniform distribution with density $\xi$, we obtain:

$$\mu_t = 1 - \Pr \left[ \varepsilon_t < \sqrt{\frac{g_t}{p_t}} \left( \frac{1}{\omega_t} - \frac{1}{\bar{\omega}_t} \right) \right] = \frac{1}{2} + \xi \sqrt{\frac{g_t}{p_t}} \left( \frac{1}{\omega_t} - \frac{1}{\bar{\omega}_t} \right).$$

(20)

Notice that if voters are surprised ($\omega_t > \bar{\omega}_t$) the incumbent increases its probability of winning over $\frac{1}{2}$. Furthermore, note that:

$$\frac{\partial \mu_t}{\partial \omega_t} = \xi \sqrt{\frac{g_t}{p_t}} \frac{1}{\omega_t^2} > 0.$$

(21)

Step 4: The incumbent’s decision in an electoral period

Taking into account $\mu_t$, the endogenous probability that the incumbent is reelected, the incumbent’s objective function is:
\[ \max E_t[c_t + \alpha \ln(g_t) + \beta(c_{t+1} + \alpha \ln(g_{t+1})) + \beta \mu_t \chi^E] \]

s.t. (3), (4), (5), (6), (16) and (20).

Incorporating these restrictions, the government’s problem in the electoral period can be reframed in terms of the choice of \( \omega_t \), that will determine all the fiscal variables:

\[ \max \{ \omega_t \geq 1 \} \]

\[ E_t \left[ y - \frac{\pi_t^*}{\omega_t} \frac{1}{\theta_t} + \alpha \ln(\theta_t + \ln \pi_t^* + \ln \omega_t) + \beta \left( y - \frac{\gamma_{t+1}^* + (1 + r)\pi_t^* (\omega_t - \frac{1}{\omega_t})}{\theta_{t+1}} + \alpha \ln(\theta_{t+1} + \ln \gamma_{t+1}^*) \right) + \beta \left[ \frac{1}{2} + \xi \tilde{\theta} \left( \frac{\omega_t}{\omega_t^*} - 1 \right) \right] \chi^E \} \]

The first order condition is given by:

\[ \frac{dE_t}{d\omega_t} \left[ \frac{\pi_t^*}{\theta_t} \frac{1}{\omega_t^2} + \frac{\alpha}{\omega_t} - \frac{\beta(1+r)\pi_t^*}{\theta_{t+1}} \left( 1 + \frac{1}{\omega_t^2} \right) + \beta \xi \tilde{\theta} \frac{\omega_t}{\omega_t^*} \chi^E \right] \leq 0, \]

with strict equality if \( \omega_t > 1 \).

which can be simplified, using the definition of \( \pi_t^* \):

\[ \frac{dE_t}{d\omega_t} = \frac{\alpha}{\omega_t^3} + \frac{\alpha}{\omega_t} - \beta(1+r) \frac{E_t \left( \frac{1}{\theta_{t+1}} \right)}{E_t \left( \frac{1}{\theta_t} \right)} \left( 1 + \frac{1}{\omega_t^2} \right) + \beta \xi \tilde{\theta} \frac{\omega_t}{\omega_t^*} \chi^E \leq 0, \]

with strict equality if \( \omega_t > 1 \). (22)

Note that \( \frac{d^2E_t}{d\omega_t^2} = -\frac{2\alpha}{\omega_t^4} - \frac{\alpha}{\omega_t^2} + 2\alpha(1+r) \frac{E_t \left( \frac{1}{\theta_{t+1}} \right)}{E_t \left( \frac{1}{\theta_t} \right)} \frac{1}{\omega_t^2} \), which it is strictly negative for \( \omega_t \geq 1 \) if the following condition holds:

\[ \beta(1+r) \frac{E_t \left( \frac{1}{\theta_{t+1}} \right)}{E_t \left( \frac{1}{\theta_t} \right)} < \frac{3}{2} \]

(23)

Assuming (23) the first order condition (22) becomes sufficient for an optimum.
People have rational expectations; so in equilibrium $\hat{\omega}_t$ must be equal to $\omega_t$. Therefore, if we call $\omega_{ue}^t$ the equilibrium value of $\omega_t$, we obtain:

$$dE_t = \frac{\alpha}{(\omega_{ue}^t)^2} + \frac{\alpha}{\omega_{ue}^t} - \beta(1 + r)\frac{E_t(\frac{1}{\eta_{t+1}})}{E_t(\frac{1}{\pi})} \left(1 + \frac{1}{(\omega_{ue}^t)^2}\right) +$$

$$+ \beta \xi \frac{1}{\omega_{ue}^t} - \chi \leq 0,$$

with strict equality if $\omega_{ue}^t > 1$.

It is clear that if there is no opportunistic motive ($\chi^e = 0$), then this expression evaluated at $\omega_{ue}^t = 1$ is negative (recall condition (8)) and the incumbent will not distort fiscal outcomes. On the other hand, with positive exogenous rents from power ($\chi^e > 0$) the above expression can become positive at $\omega_{ue}^t = 1$, which implies that the incumbent prefers a strictly positive $\omega_t$. We summarize these results in the following proposition.

**Proposition 3** With an unchecked executive, assume that conditions (8) and (23) hold, i.e., $1 < \beta(1 + r)\frac{E_t(\frac{1}{\eta_{t+1}})}{E_t(\frac{1}{\pi})} < \frac{3}{2}$. Let $\bar{\chi}_t = \frac{2\alpha}{\beta(1 + r)\frac{E_t(\frac{1}{\eta_{t+1}})}{E_t(\frac{1}{\pi})} - 1}$. Then in an electoral period ($t$ odd):

1. If $\chi^e \leq \bar{\chi}_t$ an unchecked executive does not distort fiscal outcomes ($\omega_{ue}^t = 1$)
2. If $\chi^e > \bar{\chi}_t$ an unchecked executive distorts fiscal outcomes ($\omega_{ue}^t > 1$).

**Corollary 2** Employing the expression (16) we obtain in an electoral period ($t$ odd):

1. If $\chi^e \leq \bar{\chi}_t$ then $\gamma_{ue}^t = \gamma_t^*$ and $\pi_{ue}^t = \pi_t^*$
2. If $\chi^e > \bar{\chi}_t$ then $\gamma_{ue}^t = \omega_{ue}^t \gamma_t^*$ and $\pi_{ue}^t = \frac{\pi_t^*}{\omega_{ue}^t}$

Suppose that the randomly selected and unconstrained executive $E$ must formulate optimal plans in the initial non-electoral period $t = 0$. Viewed at $t = 0$, when the government sets policy in advance, the probabilities of reelection $\mu_t$ are exogenous and equal to $1/2$ in expected value. Therefore, the government’s best policy is to plan to pick $\gamma_t^*$ and $\pi_t^*$, that is socially optimal every period, which maximizes social welfare.

The problem with this optimal plan, of course, is that it is not time-consistent: when an electoral period arrives, the government has an incentive to increase expenditure and reduce taxes. This credibility problem underlies Proposition 3 under an unchecked executive.
What happens if the status quo is set according to rule (11)-(12)? Well, if the rule were binding, this would effectively curb the credibility problem: in an electoral period the executive would prefer to use debt to increase expenditures and reduce taxes in order to look more competent, but the status quo rules out public indebtedness. However, it does not make sense to assume that the executive is constrained to follow any rule, unless it has to share the power to change rules with another body. Otherwise, if the executive is also vested with legislative power, it can do and undo any rule it likes, being effectively unconstrained. The natural environment where the executive shares rule-making power is when there is divided government, and an agreement has to be reached with the veto player $L$ on changes in the budget.

### 3.3 Separation of powers

We now turn to fiscal policy under separation of powers. We distinguish between divided and unified government. For both presidential and parliamentary systems, we describe divided government in terms of $E$ being in the hands of one party and $L$ in the hands of the other.\(^5\)

Suppose that in period $t = 0$ the randomly selected government is a unified one. Without loss of generality, assume that party $A$ control both the executive and the legislature. Let debt $d_{-1} \geq 0$. Since we have an aligned legislature or a single party government, nobody will veto proposals by $E$. This implies, by Proposition 3, that with sufficiently large opportunism there will be an electoral cycle in fiscal policy in $t = 1$, the first electoral period. In period $t = 0$ party $A$ does not have any incentive to distort fiscal variables. Hence, it just selects optimal expenditures and repays past debt, if any.

As regards voters, at election in $t = 1$ they will want the party with the highest expected competency holding the executive, just as in the case of an unchecked executive. At the same time, we conjecture they will want to have divided government. This is so, because in terms of government competence it is indifferent for voters whether a single party controls both the executive and the legislature, or if two parties share control. But in terms of the distortion of fiscal variables, divided government is strictly preferred if an opposition legislature can block the executive’s attempts to distort the budget in period $t = 3$ to look more competent. Putting all this together, we deduce that voters will prefer to vote for divided government in period $t = 1$.

Does what actually happens in periods $t = 2$ and $t = 3$ under divided government conform to these conjectures? Let us assume that $A$ controls

---

\(^5\)Saporiti and Streb (2004) consider separation of powers with a randomly elected legislature that represents the interests of the people, so the legislature is never aligned with the executive. Here, the issue of unified or divided government is endogenous and depends on voters.
the executive and B the legislature. In the electoral period $t = 3$, the executive would like to increase its electoral chance by using debt to select $\pi^e_3$ and $\gamma^e_3$. However, party B can veto this and any attempt of A to employ debt to increase expenditures and reduce taxes, since the status quo debt restriction given by (12), i.e., $d_3 \leq d_2$, introduces an effective restriction in the executive’s opportunities. Party B has the motivation and the power to veto any attempt of party A to use debt to increase its electoral chances. Therefore, party A is forced to set expenditures equal to taxes. Given that it cannot affect its perceived competency, the best party A can do is to select an optimal level of taxes and expenditures. Notice also that, should the legislature veto these optimal level of taxes and expenditures, this would not affect the voters’ perception of party A’s competence, since what voters use in their inference problem is the no new debt restriction, which implies that $\gamma_3 = \pi_3$, so the ratio $g_t/p_t$ can be used to infer competency whatever the level of expenditure. Given this, the legislature has no incentive to block optimal expenditure in election periods.

As to the non-electoral period $t = 2$, the executive, controlled by party A, chooses an optimal expenditure and repays past debt, if any, because whatever it does then does not affect its electoral chances in the next electoral period, only current welfare. The legislature, controlled by party B, does not want to veto this proposal, because if does not affect future reelection prospects of either party, and lead to optimal outcome in non-electoral period. This confirms the voters’s conjectures we assumed at the outset.

Putting together the arguments of the last three paragraphs, and extending the logic to all future time periods, we get the following conclusion:

**Proposition 4** Suppose there is separation of powers and the legislature must authorize new debt. Under perfect compliance with the budget law, there is no electoral cycles, except in the first electoral period if we begin with a unified government.

### 3.4 Imperfect compliance

At the implementation stage, $E$ supplies the public goods, but it is monitored by $L$. We now introduce a measure of the effective compliance with the balanced budget rule. Either there is perfect compliance, or imperfect compliance: $\lambda \in \{0, 1\}$. The measure $\lambda$ can be interpreted as the degree of compliance with the authorized budget, and describes the effective limits $L$ imposes on the executive office. If there is no compliance with the budget law, divided government is useless: the legislature cannot check the electoral manipulation of the budget. Hence, there is no incentive to have divided government.

This proposition implies a sharp (and falsifiable) prediction: PBC should be present in countries with imperfect compliance with the law. These
countries should also have less incidence of divided government. This can be empirically related to the evidence on the existence of stronger cycles in developing countries, where there is typically less compliance with the rule of law than in developed countries (Streb, Lema, and Torrens 2007 study these empirical implications).

4 Shared government

An extension of the model is to combine the agenda setter model of Romer and Rosenthal (1978, 1979), where the policy-making process carried out to set the budget can be reduced to the interaction of the current leaders of the two branches of government, the executive and the legislature, with a stylized model of government performance when parties share power.

4.1 Government competence

We assume that under separation of powers the competence of the government depends on the competence of both the executive and legislative branches, $E$ and $L$:

$$
\theta_t = \rho \theta_t^E + (1 - \rho) \theta_t^L, \text{ for } \rho \in (1/2, 1],
$$

(24)

where $\rho > 1/2$ to capture the characteristic that the executive has the main responsibility for government performance, though unlike the earlier literature on PBC, government competence need not be identical with that of the executive.

The competence of the two branches of government depend in turn on the competence of the two political parties $A$ and $B$ in charge,

$$
\theta_t^E = \begin{cases} 
\theta_t^A & \text{if party (leader) } A \text{ control the executive} \\
\theta_t^B & \text{if party (leader) } B \text{ control the executive}
\end{cases},
$$

(25)

$$
\theta_t^L = \begin{cases} 
\theta_t^A & \text{if party (leader) } A \text{ control the legislative} \\
\theta_t^B & \text{if party (leader) } B \text{ control the legislative}
\end{cases},
$$

(26)

and

$$
\theta_t^A = \bar{\theta} + \varepsilon_{t-1}^A + \varepsilon_t^A, \quad \text{(27)}
$$

$$
\theta_t^B = \bar{\theta} + \varepsilon_{t-1}^B + \varepsilon_t^B. \quad \text{(28)}
$$

For each party $i = A, B$, the variable $\varepsilon^i$ is a random i.i.d. variable which is uniformly distributed over the interval $[-\frac{1}{2\xi}, \frac{1}{2\xi}]$, with expected value $E(\varepsilon) = 0$ and density function $\xi > 0$. A higher value of
Corresponds to a more competent politician. The probability distribution of $\theta_i$ conditional on $\varepsilon_i$, $F(\theta_i | \varepsilon_i)$, is also uniform, with support $[\bar{\theta} + \varepsilon_i - \frac{1}{2\xi}, \bar{\theta} + \varepsilon_i + \frac{1}{2\xi}]$, and $E(\theta_i | \varepsilon_i) = \bar{\theta} + \varepsilon_i$.

Replacing the competence of parties in charge of $E$ and $L$ in (7) leads to

$$\theta_t = \bar{\theta} + \left( \rho \varepsilon_{i-1}^j + (1 - \rho) \varepsilon_{i-1}^j \right) + \left( \rho \varepsilon_{t}^i + (1 - \rho) \varepsilon_{t}^i \right).$$

(29)

Replacing $\varepsilon = \rho \varepsilon^i + (1 - \rho) \varepsilon^j$, with $i, j \in \{A, B\}$, leads to equation (29), where actual competence of the government is partially lasting, following a first-order moving average process as in Rogoff and Sibert (1988) and others:

$$\theta_t = \bar{\theta} + \varepsilon_{t-1} + \varepsilon_t.$$  

Henceforth, $\bar{\theta} > 1/\xi$, so $\theta_t > 0$ and (5) and (6) are well-defined.

4.2 Signal extraction problem

Since there are two parties, $A$ and $B$, whose leaders can control $E$ and $L$, government can be as follows:

<table>
<thead>
<tr>
<th>Parties in government</th>
<th>Leader of $E$</th>
<th>Leader of $L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) $A, A$</td>
<td>$A$</td>
<td>$A$</td>
</tr>
<tr>
<td>(ii) $A, B$</td>
<td>$A$</td>
<td>$B$</td>
</tr>
<tr>
<td>(iii) $B, A$</td>
<td>$B$</td>
<td>$A$</td>
</tr>
<tr>
<td>(iv) $B, B$</td>
<td>$B$</td>
<td>$B$</td>
</tr>
</tbody>
</table>

In cases (i) $A, A$ and (iv) $B, B$, government competence reflects the competence of the sole party in charge of government. Given a uniform distribution, equal competence has zero probability, and voters will strictly prefer either the incumbent or the opposition party.

In cases (ii) $A, B$ and (iii) $B, A$, to determine the competence of the parties in charge of running government, the voters face a signal extraction problem. Without loss of generality, let $A$ be in $E$ and $B$ in $L$ (the converse case is analogous). Hence, $\varepsilon^A$ represents the shock to party that runs $E$, and $\varepsilon^B$ the shock to the party that runs $L$. Given $\rho > 1/2$, if there is a positive shock the conditional probability that incumbent of $E$ has high competence will be larger than the conditional probability that incumbent of $L$ has high competence, so based solely on competence citizens will want to vote for party in charge of $E$ for both roles (we liken political party to party leader: if $E$ and $L$ are in hands of same party, competence of party leader rules performance in both branches). In case of negative shocks, the vote will favor the leader of $L$, that is opposition.

With a uniform distribution, the signal extraction problem has a simple analytic solution. As to the conditional probability $E$ is competent, since...
the distributions of $\varepsilon^A$ and $\varepsilon^B$ are symmetric and independent, the sum of both shocks is positive half the times. This forms the denominator of the conditional probability. As to the numerator, one fourth of the time, both shocks are positive, so the sum is positive. One fourth of the time, the shocks $\varepsilon^A$ are positive, while the shocks to $\varepsilon^B$ are negative; the sum is positive if $\rho \varepsilon^A + (1 - \rho)\varepsilon^B > 0$, which is the case whenever $|\varepsilon^A| > (1 - \rho)/\rho$, with probability $(1 - (1 - \rho)/\rho)$ because of uniform distribution; when this condition is not met, this is true only half the time, $(1/2)(1 - \rho)/\rho$, again because of uniform distribution. Hence, with a positive shock, the conditional probability $A$ is competent is

$$P(\varepsilon^A > 0|\varepsilon > 0) = \frac{1/4(1 + (1 - \rho)/\rho + (1 - \rho)/2\rho)}{1/2},$$

(30)

so if $\rho = 1/2$, $P(\varepsilon^A > 0|\varepsilon > 0) = 3/4$, while if $\rho = 1$, $P(\varepsilon^A > 0|\varepsilon > 0) = 1$, with $dP/d\rho = 1/2(1/\rho^2) > 0$.

As to the conditional probability $B$ is competent, both shocks are positive one fourth of the time, so sum is positive; one fourth of the time, the shocks to $\varepsilon^A$ are negative and the shocks to $\varepsilon^B$ are positive, and the sum is only positive half the time that $|\varepsilon^A| < (1 - \rho)/\rho$:

$$P(\varepsilon^B > 0|\varepsilon > 0) = \frac{1/4(1 + (1 - \rho)/2\rho)}{1/2}.$$  

(31)

Thus, if $\rho = 1/2$, $P(\varepsilon^B > 0|\varepsilon > 0) = 3/4$, so government performance leads to the same expected competence for both parties, while if $\rho = 1$, $P(\varepsilon^B > 0|\varepsilon > 0) = 1/2$, so we learn nothing about the competence of the opposition party in charge of the legislature from government actions as in the standard PBC models. Here, $dP/d\rho = -1/2(1/\rho^2) < 0$.

Our presentation has been done in terms of the leaders of the two branches of government, $E$ and $L$. However, equation (29) carries over from a presidential system to a parliamentary system, where the players are the major and minor members of the government coalition. Cases (i) and (iv) in Table 1 now refer to a single party governments, while cases (ii) and (iii) refer to a coalition government were there is a major partner and a minor partner. Since coalition governments divide cabinet posts according to the importance of the members of ruling coalition, it is immediate to put the largest share of responsibility for government performance on the major partner.

4.3 Voting decision

What does not translate so easily from a presidential system to a parliamentary system is the voting decision that leads to governments in Table 1. In a presidential system, the representative voter has two separate votes, and can decide whether to support the same party in the executive branch and the
legislature. In a parliamentary system, an individual voter cannot literally split its vote among the two political parties, since there is no separate vote for the legislature. However, the representative voter has a preference for whether it wants only one party to run the government, or whether it wants two parties to share power. If we allow for fictitious vote splitting, allowing the representative voter to split its vote in a given proportion between parties $A$ and $B$, this can artificially recreate what the electorate at large can do, with a certain proportion voting one party and another proportion voting the other. With our representative (median) voter who can split votes, we are skipping over the need to coordinate votes among the electorate at large, and the specific process by which certain vote totals lead either to a single party or to a coalition government. Our purpose at hand is whether only one or more than one party run the government.

5 Conclusions

The fact that the executive incumbent is unable to credible compromise to the optimal allocation policy is at the heart of these electoral distortions. Furthermore, it turns out that this problem is in fact generated by concentration of powers, which allows the executive to choose any policy it desires. Instead, when there exists separation of powers, appropriate checks and balances work as a commitment device that reduces the size of electoral fiscal cycles, making all players better off (including the executive incumbent). With an exogenous status quo, this moderating force depends on the details of the bargaining game, namely the exact status quo location, the actual agenda-setting authority and the degree of compliance with the budget law. With an endogenous status quo given by the previous period’s budget, the predictions are a lot simpler: separation of powers eliminates PBC, unless there is a low degree of compliance with the approved budget.

More generally, in relation to the debate on rules versus discretion, our discussion of PBC shows that a way to solve the credibility problem, making the budget rule a credible commitment, is to introduce an institutional arrangement of separation of powers that limits the discretion to change rules. Even though we do not consider signaling models of PBC à la Rogoff, it should be expected that separation of powers affect electoral fiscal cycles in a similar way. The legislature basically tries to avoid distortions in the allocation of budget resources. This should reduce the electoral distortions of fiscal policy, preserving the signaling role of the provision of public goods.\footnote{Notice that in models of PBC à la Rogoff, the timing of events is reversed in relation to Lohmann. That is, the incumbent observes its competence before choosing the period policy, not afterwards. However, the informativeness of the signal is not larger in equilibrium, since there is a separating equilibrium with both types of models. Besides, the Rogoff timing brings in an extra complication. The signal depends on the incumbent’s type, something that is not required to explain the policy bias in electoral periods.}
Our results are derived in a stationary environment where the optimal allocation of the budget is constant over time. In a stochastic environment, one can conjecture that the budget rule we analyze may still be optimal if shocks to the desired budget allocation follow a random walk. What may change the results more fundamentally is lifting the assumption that the legislature has no electoral stakes. In this regard, our case is the best scenario where the legislature controls the executive to try to assure the socially optimal policy is followed.

Our model might be extended to study the role of divided government in presidential systems, as well as coalition governments in parliamentary systems. For instance, Alesina and Rosenthal (1995) show how divided government is a tool to moderate the executive in a presidential system. A similar logic may apply in an opportunistic framework, where an opposition legislature may play a special role in moderating cycles. Finally, our model of PBC under separation of powers could also be employed to understand how the incumbent chooses among different fiscal instruments or why it uses some of them more frequently in some countries than in others. Even though fiscal policy includes several items, like taxes, expenditure and debt, there is no general model of rational PBC that explains how politicians choose between these instruments. Following the logic of our model, it should be expected that institutional details play an important role in the selection. This is because the executive should manipulate those fiscal instruments where it has greater agenda-setting authority. It is left for a future research to formally explore this conjecture, as well as its empirical validity.

This may be empirically relevant, since Alesina, Roubini, and Cohen (1997, chaps. 4 and 6) trace the lack of recent evidence on opportunistic cycles in the United States back to the fact that after 1980 many federal transfer programs have become mandatory by acts of Congress, so they cannot be easily manipulated for short run purposes.

6 Appendix A: Proof of Proposition 2

We will prove this proposition in two steps. First, we will prove that the expected value of a function of two stochastic independent variables is equal or greater than the expected value of the same function, conditional on the realization of a third variable that generates an estimation of one of the variables, if and only if the function is increasing and concave. Secondly, we will prove that $u(c, g)$, considered as a function of the two independent stochastic variables $\varepsilon_t$ and $\varepsilon_{t-1}$ is increasing and concave. Moreover, it has the unappealing implication that competent incumbents distort the most, while the utterly incompetent incumbents do not (Streb 2003 shows how heterogeneity in opportunism can overcome this feature).
Lemma 1  Let $Z = h(X,Y)$ be a function of two independent stochastic variables $X$ and $Y$, with marginal densities $f_x(x)$ and $f_y(y)$. Let us call $g(x) = \mathbb{E}[Z \mid x]$ the expected value of $Z$ conditional on $x$. Consider a known vector of information variables $W$ that allows to estimate $X$ and call $\hat{x}(w)$ the calculated value of $X$ when $W$ adopts the value $w$. Suppose that $g(x)$ is an increasing and concave function of $x$. Then

\[ \mathbb{E}[Z \mid \hat{x}(w)] \geq \mathbb{E}[Z] \text{ if and only if } \hat{x}(w) \geq \mathbb{E}[X]. \]

Proof 2  First notice that since $X$ and $Y$ are independent stochastic variables $g(x) = \mathbb{E}[Z \mid x] = \int h(x,y) f_y(y) \, dy$. Since $g(x)$ is concave, by Jensen’s inequality it follows that $g(\mathbb{E}[X]) \geq \mathbb{E}[g(x)]$. Employing the definition of $g$ the first term of the inequality is just $\mathbb{E}[Z \mid \mathbb{E}[X]]$, while the second term is equal to $\mathbb{E}_X[\mathbb{E}[Z \mid X]]$. Therefore, we get $\mathbb{E}[Z \mid \mathbb{E}[X]] \geq \mathbb{E}_X[\mathbb{E}[Z \mid X]]$. By the law of iterated expectations $\mathbb{E}[Z] = \mathbb{E}_X[\mathbb{E}[Z \mid X]]$. Hence,

\[ \mathbb{E}[Z \mid \hat{x}(w)] \geq \mathbb{E}[Z] \quad (32) \]

Now, consider the vector of information variables $W$, whose realization $w$ is known. It is clear from inspection of (32) that if $g(x) = \mathbb{E}[Z \mid x]$ is an increasing function of $x$, then $\mathbb{E}[Z \mid \hat{x}(w)] \geq \mathbb{E}[Z \mid \mathbb{E}[X]]$ if and only if $\hat{x}(w) \geq \mathbb{E}[X]$. Therefore, $\mathbb{E}[Z \mid \hat{x}(w)] \geq \mathbb{E}[Z]$ if and only if $\hat{x}(w) \geq \mathbb{E}[X]$.

In our case the two stochastic independent variables are $\varepsilon_t$ and $\varepsilon_{t+1}$, the vector of information variables is integrated by $g_t$, $p_t$, and $\tilde{\omega}_t$, and $h(\varepsilon_t, \varepsilon_{t+1}) = c_{t+1} + \alpha \ln(g_{t+1})$. It remains to prove that $\mathbb{E}_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]$ is an increasing and concave function. We begin using expressions ((3), (4), (5), (6)) to replace $c_{t+1}$ and $g_{t+1}$ (line 1). Next we replace $\gamma^{we}_t$ and $\pi^{we}_t$ for their respective values (lines 2 and 3). Finally in lines 4 and 5 we apply the conditional expected value operator.
\[ E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t] = E_t[y - \frac{\pi_{\varepsilon t_{t+1}}^{\mu c}}{\theta_{t+1}} + \alpha \ln(\theta_{t+1}^{-1}_{\varepsilon t_{t+1}}) \mid \varepsilon_t] = \]

\[ = E_t[y - \frac{\alpha}{\xi \ln(\theta_{t+1}^{\varepsilon t_{t+1}})} + (1 + r)d_t + \alpha \ln(\varepsilon_{t+1} + 1) \mid \varepsilon_t] = \]

\[ + \alpha \ln(\bar{\theta} + \varepsilon_t + \varepsilon_{t+1}) + \alpha \ln \left( \frac{\alpha}{\xi \ln(\theta_{t+1}^{\varepsilon t_{t+1}})} \right) \mid \varepsilon_t] = \]

\[ = y - \alpha - (1 + r)d_t \xi \ln \left( \frac{\bar{\theta} + \varepsilon_t + \frac{1}{2\xi}}{\theta + \varepsilon_t - \frac{1}{2\xi}} \right) + \]

\[ + \alpha \ E_t[\ln(\bar{\theta} + \varepsilon_{t+1} + \varepsilon_t) \mid \varepsilon_t] + \alpha \ln \left( \frac{\alpha}{\xi \ln(\theta_{t+1}^{\varepsilon t_{t+1}})} \right) \]

The last expression is increasing in \( \varepsilon_t \); a fact that can be confirmed deriving it with respect to \( \varepsilon_t \):

\[ \frac{\partial E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]}{\partial \varepsilon_t} = \frac{(1 + r)d_t}{(\bar{\theta} + \varepsilon_t)^2 - \frac{1}{4\xi^2}} + \]

\[ + \alpha \xi \ln \left( \frac{\bar{\theta} + \varepsilon_t + \frac{1}{2\xi}}{\theta + \varepsilon_t - \frac{1}{2\xi}} \right) + \alpha \xi \left[ (\bar{\theta} + \varepsilon_t)^2 - \frac{1}{4\xi^2} \right] \ln \left( \frac{\theta + \varepsilon_t - \frac{1}{2\xi}}{\theta + \varepsilon_t - \frac{1}{2\xi}} \right) > 0. \]

\( E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t] \) is also concave in \( \varepsilon_t \), which is true for \( \alpha \) sufficiently low.

\[ \frac{\partial^2 E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]}{\partial \varepsilon_t^2} = (-1) \frac{(1 + r)d_t 2(\bar{\theta} + \varepsilon_t)}{\left[(\bar{\theta} + \varepsilon_t)^2 - \frac{1}{4\xi^2}\right]^2} + \]

\[ - \frac{\alpha}{\left[(\bar{\theta} + \varepsilon_t)^2 - \frac{1}{4\xi^2}\right]} - \frac{\alpha}{\xi \left[(\bar{\theta} + \varepsilon_t)^2 - \frac{1}{4\xi^2}\right] \ln \left( \frac{\theta + \varepsilon_t + \frac{1}{2\xi}}{\theta + \varepsilon_t - \frac{1}{2\xi}} \right)} < 0. \]

References


