Human Capital Investment and Progressive Income Taxation *

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June 22, 2007

Abstract

This paper assesses the impact of progressive income taxation on human capital accumulation. Using a quantitative version of the Ben-Porath (1967) human capital model and a partial equilibrium life-cycle model, it is found that progressivity has a negative impact on human capital accumulation. In particular, high ability agents are the most affected by progressive income taxation. However, reducing progressivity increases income inequality in the economy, making apparent an “efficiency-equality” trade-off.

JEL Classification: D3, H2, J24, J31.

Keywords: Progressive Income Taxation, Human Capital, Earnings Distribution, Heterogeneous Agents.

*I would like to thank my supervisor, Josep Pijoan-Mas for his great help and advises. I am grateful to Gustavo Ventura for providing me estimates for the PSID data. Finally, I would like to thank Emilia Balagué, Paula Papp, Julián Pasin and Sebastián Rondeau for helpful comments and support. The usual disclaimer applies.

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1 Introduction

Tax schedule structure is a subject under continuous discussion in the economic literature and in politics. Given the high concentration of income and wealth in the U.S. economy and the prevalence of progressive income taxes, it seems relevant to study the effects of tax progressivity.\footnote{Cagetti and De Nardi (2006) summarize some key facts about the wealth distribution and the economic models that have been used to explain it.} \footnote{See, for example, Petska and Strudler (2000).}

Several works have studied the impact of tax progressivity on human capital accumulation, labor supply and growth under different frameworks. For instance, Ziliak and Kniesner (1999) find that recent U.S. tax reforms that produced a flatter income tax stimulated labor, reduced dead weight loss and were not self-financing. Under the context of heterogeneous agents in an overlapping generations model, Heckman et al. (1998b) analyze the effects of two tax regimes on skill formation focusing on time variation in the skill premium. They conclude that changes from progressive to proportional taxation are unlikely to have large effects on skill formation or output. On the other hand, Caucutt et al. (2003) and Caucutt et al. (2006) find that a less progressive tax system gives rise to increased growth, increased investment in human capital, decreased inequality and greater mobility for the poor in the long run. However, Li and Sarte (2004) suggest that the Tax Reform Act of 1986, which reduced progressivity, produced a small increase in the long-run growth but had significant effects on increasing income inequality. Thus, these contradicting claims regarding the connection between efficiency and equality make the analysis under an alternative heterogeneous agent model appealing.

In order to evaluate the distortions associated to progressive income taxation and the change in the income distribution, the heterogeneous agent framework developed by Huggett et al. (2006) is extended. Analyzing how average earnings and measures of earnings dispersion and skewness change for a typical cohort of individuals as the cohort ages, Huggett et al. (2006) develop an alternative version of the Ben-Porath (1967) human capital model, in which each agent is endowed with some immutable learning ability and some initial human capital. There are several appealing reasons to use this model in order to analyze the impact on human capital investment. First, the model is able to replicate the key statistics of the earnings distribution depicted in Figure 1. Second, wages are endogenous in the model as they are defined as the product of a rental rate, human capital and time allocated to work. As earnings profiles are determined by the optimal investment of time and resources into the accumulation of skills, this implies that investment decisions will not be invariant to changes in government policies. Third, one of the main results of the model is that at the end of the working life cycle the agents who are high earners are precisely those who started off with high initial human capital and/or ability. As the model endogenizes wage differences via human capital theory, if the government taxes current income in
order to decrease inequality, it will be penalizing the most productive agents of the economy. This would mean that these agents would invest less in human capital formation, thus reducing the human capital stock of the economy. To include a tax schedule to the cited model, an “effective tax function” as developed by Gouveia and Strauss (1994) is incorporated. Besides providing a close approximation to the actual U.S. income tax code, the flexibility of this functional form allows nesting a proportional tax code, a wide variety of progressive and regressive tax codes and more important, it makes numerical approximation over the income tax code feasible.

The main finding of this paper is that progressivity has a negative impact on human capital accumulation. Besides, a trade-off becomes apparent as reducing progressivity increases income inequality. In particular, a shift from the benchmark progressive schedule to a proportional one, causes an increase of 10.65% in time invested in human capital accumulation and of 8.31% in after-tax earnings for the economy in average, while the Gini coefficient raises 25.21%. By reducing progressivity, agents in the economy devote more time investing in human capital accumulation, producing a higher aggregate level of human capital and increasing the present value of earnings for the economy mean. For instance, agents endowed with high ability at birth have steeper sloped age-earnings profiles than low ability agents, other things equal. This is due to the fact that early in life high ability agents allocate a larger fraction of their time investing in human capital, making that later in the life-cycle they have higher levels of human capital and, hence, earnings. By increasing progressivity, these agents reduce significantly time invested early in their lives as at the end of their lives they will face higher marginal taxes, provoking a decrease in their present value of after-tax earnings. The aforementioned decrease induced by a progressive schedule produces a fall in earnings dispersion and skewness and a reduction in human capital accumulation.

The paper is organized as follows. Section 2 describes the model. Section 3 describes the data and the empirical methodology. Section 4 explains how model parameters are set. Section 5 presents the central results produced by the model. Section 6 concludes.

2 The model

2.1 Human capital theory

An agent decides the time spent in human capital production and market work in order to maximize the present value of after-tax earnings over the working lifetime. In the absence of a labor-leisure decision and liquidity constraints, utility maximization implies present value after-tax earnings maximization.\(^3\) The deci-

\(^3\)Consumption and asset choice over the life-cycle are abstracted from this model, and so any substitution effect that the tax schedule may cause. However, these effects are at most modest for the study of human capital accumulation and the implied labour earnings dynamics.
sion problem formulated in the language of dynamic programming is the one given in (1) below.

The value function $V_j(h, a)$ gives the maximum present value of after-tax earnings at age $j$ from state $h$ when learning ability is $a$. The value function is set to zero after the last period of life. Solutions to this problem are given by optimal decision rules $h_j(h, a)$ and $l_j(h, a)$. Solutions to this problem are given by optimal decision rules $h_j(h, a)$ and $l_j(h, a)$ which describe the optimal choice of human capital carried to the next period and the fraction of time spent in human capital production as functions of age $j$, human capital $h$ and learning ability $a$.

\[
V_j(h, a) = \max_{l, h'} \left\{ e - T(e) + (1 + r)^{-1}V_{j+1}(h', a) \right\}
\]

s.t. : $l \in [0, 1]$, $h' = h(1 - \delta) + f(h, l, a)$, \hspace{1cm} (1)

where $r$ is a real interest rate and pre-tax earnings $e$ in a period equal the product of the rental rate of human capital $w_j$, the agent’s human capital $h$ and the time spent in market work $(1 - l)$. $T(e)$ denotes total taxes paid with pre-tax income $e$. The stock of human capital increases when human capital production offsets the depreciation of current human capital $\delta$. Human capital production $f(h, l, a)$ depends on an agent’s learning ability $a$, human capital $h$ and the fraction of available time $l$ put into human capital production. Learning ability is fixed at birth and thus does not change over time.

### 2.2 Tax function

In this model the function of the government is to levy taxes and use income tax revenues to finance a fixed level of expenditures. The functional tax form used is the theoretically motivated by the equal sacrifice principle and is fairly flexible in that it encompasses a wide range of progressive, proportional and regressive tax schedules.\(^4\) Letting $T(e)$ denote total taxes paid with pre-tax income $e$, the tax code is restricted to the functional form:

\[
T(e) = \tau_0 \left[ e - \left( e - \tau_1 + \tau_2 \right)^{-\frac{1}{\tau_1}} \right], \hspace{1cm} (2)
\]

where $(\tau_0, \tau_1, \tau_2)$ are parameters.

Note that the limiting marginal and average tax rate equals $\tau_0$. For $\tau_1 = -1$, the scheme is a constant tax independent of income: $T(e) = -\tau_0 \tau_2$. For $\tau_1 \to 0$ the scheme is the purely proportional system: $T(e) = \tau_0 e$. Finally, for $\tau_1 > 0$ the scheme is progressive since average (as well as marginal) taxes are a strictly increasing function of income $e$:

\[
t(e) = \frac{T(e)}{e} = \tau_0 \left[ 1 - (1 + \tau_2 e^{\tau_1})^{-\frac{1}{\tau_1}} \right], \hspace{1cm} (3)
\]

\[
T'(e) = \tau_0 \left[ 1 - (1 + \tau_2 e^{\tau_1})^{-\frac{1}{\tau_1} - 1} \right]. \hspace{1cm} (4)
\]

\(^4\)See Gouveia and Strauss (1994).
As function \( (2) \) was estimated for incomes in 1989 U.S. dollars and \( \tau_2 \) is not invariant to units of measurement, the tax formula is normalized so as to equalize the percentage of labour income levied by the government in the U.S. economy and in the model.

2.3 Model results\(^5\)

**Proposition 1** Assume the human capital production function is given by \( f(h, l, a) = a(hl)^{\alpha} \), \( \alpha \in (0, 1) \), the depreciation rate \( \delta \in [0, 1) \), the rental wage equals \( w_j = (1 + g)^{j-1} \) and the gross interest rate \( (1+r) \) is strictly positive. Let \( \eta \equiv \frac{1}{a}[h' - (1-\delta)h] \) and \( \eta' \equiv \frac{1}{a}[h'' - (1-\delta)h'] \). Then,

(a) \( V_j(h, a) \) is continuous and increasing in \( h \) and \( a \), is concave in \( h \) and \( h_j(h, a) \) is single-valued.

(b) The decision rule for time invested \( l_j(h, a) \) is given by:

\[
l_j(h, a) = \left( \frac{h' - (1-\delta)}{\alpha} \right)^{\frac{1}{\alpha}} \equiv \eta \frac{1}{h}. \tag{5}\]

(c) The decision rule for human capital \( h_j(h, a) \) is the solution to the Euler Equation:

\[
\frac{\eta \frac{1}{\alpha}}{a\alpha} \left\{ w_j \left[ -1 + \tau_0 \left( 1 - \left\{ 1 + \tau_2 \left[ w_j \left( h - \frac{1}{\alpha} \right) \right]^{\tau_1} \right\}^{-\frac{1+\tau_1}{\tau_1}} \right) \right] + \\
+ \left\{ w_{j+1} \left[ 1 - \tau_0 \left( 1 - \left\{ 1 + \tau_2 \left[ w_{j+1} \left( h' - \frac{1}{\alpha} \right) \right]^{\tau_1} \right\}^{-\frac{1+\tau_1}{\tau_1}} \right) \right] \right\} \right. \\
\left. \cdot \left[ a\alpha + (1-\delta) \eta \frac{1}{\alpha} \right] \right\} = 0. \tag{6}\]

**Proof.** See appendix A.1. \( \blacksquare \)

Several implications follow from Proposition 1. In particular, an agent spends all time in human capital accumulation provided that current human capital is below an age and ability dependent cutoff level. Then, once an agent with ability \( a \) stops full-time schooling, the agent never returns to full-time schooling. The model also implies that late in the working life cycle human capital investments are approximately zero.

\(^5\)As the difference between the model used in this paper and the one in Huggett et al. (2006) is the addition of the “effective tax function”, the methodology followed is similar to the aforementioned paper’s one. It will be shown that qualitative results remain the same.
Moreover, the fact that the decision rule for human capital $h_j(h, a)$ is increasing in both current human capital and learning ability implies that at the end of the working life cycle the agents who are high earners are precisely those who started off with high initial human capital and/or ability. This is true since at the end of the life cycle earnings are proportional to human capital. Similar reasoning implies the greater the dispersion in earnings at the end of the working life cycle the greater is the required dispersion in human capital or learning ability at the beginning of the life cycle.

Finally, as the value of the elasticity parameter in the human capital production function is less than one, if all agents have the same ability $a$, initial human capital heterogeneity is not sufficient for producing the dispersion in earnings mentioned before. As agents with higher human capital spend less time in human capital production compared to agents with lower amounts of human capital, agents with the lower human capital have higher human capital growth rates and agents with lower earnings have higher earnings growth rates. Then, differences in learning ability are absolutely fundamental for this model to be able to produce growing earnings dispersion.

3 Data and empirical methodology

3.1 Data

The data source used in this paper is the Michigan Panel Study of Income Dynamics (PSID), a longitudinal survey which follows a sample of U.S. households from the civilian population since 1968. Approximately 5,000 households were interviewed in the initial year of the survey, including a core random sample of about 3,000 households (the SRC subsample) and a supplementary low-income sample of around 2,000 households (the Census Bureau’s SEO subsample). Members of the original sample and all their offspring are included in the dataset.

Age profiles are based on earnings data from the 1969-1992 family files. The baseline sample is restricted in the following way: males who are head of household in the core sample (the SEO subsample is excluded from the analysis), aged between 20 and 58 and having strictly positive earnings. Among these individuals, those who are currently working, temporarily laid off, looking for work but are currently unemployed and students are included. This set of requirements has been chosen for two reasons. First, the PSID has many observations in the middle but relatively fewer at the beginning or end of the working life cycle. By focusing on ages 20-58, there are at least 100 observations in each age-year bin with which to calculate age and year-specific earnings statistics. Second, near the traditional retirement age there is a substantial fall in labor force participation that occurs for reasons that are abstracted from in the model. This suggests the

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6This section follows the methodology explained in Huggett et al. (2006) and in Huggett et al. (2007).
use of a terminal age that is earlier than the traditional retirement age.

3.2 Age Profiles

Let $e_{j,t}^p$ be the real earnings at percentile $p$ of the earnings distribution of agents who are age $j$ at time $t$.\(^7\) Earnings data can be viewed as being generated by several factors such as cohort ($\alpha_s^p$), time ($\gamma_t^p$) and age ($\beta_j^p$) effects and shocks ($\epsilon_{j,t}^p$). The relationship between these variables in levels is given in equation (7) and in logarithms in equation (8). A cohort is denoted as $s = t - j$. Cohort effects can be viewed as effects that are common to all agents who were born in a particular year, while time effects can be viewed as effects that are common to all individuals alive at a point in time.

$$e_{j,t}^p = \alpha_s^p \cdot \gamma_t^p \cdot \beta_j^p \cdot \epsilon_{j,t}^p ,$$

(7)

$$\tilde{e}_{j,t}^p = \tilde{\alpha}_s^p + \tilde{\gamma}_t^p + \tilde{\beta}_j^p + \tilde{\epsilon}_{j,t}^p .$$

(8)

The analysis focuses on age effects as it provides a clean measure of earnings and inequality per age which the model should match. However, this measure depends on the identifying assumptions regarding cohort and time effects. As the linear relationship between time $t$, age $j$, and birth cohort $s$ limits the applicability of the regression specification above, without further restrictions the regressors in this system are collinear and these effects cannot be estimated. In effect, any trend in the data can be arbitrarily reinterpreted as a year (time) trend or alternatively as trends in ages and cohorts.

In order to solve this problem, time effects are set to 0 (i.e. $\tilde{\gamma}_t^p = 0 \forall t$).\(^8\) Ordinary least squares is used to estimate the coefficients. The regression has $J \times T$ dependent variables regressed on $J + T$ cohort dummies and $J$ age dummies. $T$ and $J$ denote the number of time periods in the panel and the number of distinct age groups, which are $J = 58 - 20$ and $T = 1992 - 1969$. Then age effects $\beta_j^p$ are scaled so that mean before-tax earnings equal 100 at the end of the working life cycle.

Figure 1(a) shows that average earnings increase with age over most of the working life cycle. Early in the life cycle this follows because earnings at all percentiles shift up with age. Later in the life cycle this follows from the strong increase with age at the highest percentiles of the earnings distribution despite the fact that earnings at the median and lower percentiles are already decreasing with age. The increase in earnings dispersion in Figure 1(b), using the Gini coefficient as a measure of earnings dispersion, follows from the general fanning out of the distribution. The increase in the skewness measure with age in Figure 1(c) is

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\(^7\)Real values are calculated using the CPI. A 5-year bin centered at age $j$ is used to calculate $e_{j,t}^p$.

\(^8\)This is the so-labeled cohort dummies view. See Huggett et al. (2006) and Huggett et al. (2007) for the econometric discussion.
implied by the strong fanning out at the top of the distribution. In section 5.4 we will analyze which are the explanations the model can give to these earnings facts.

4 Methodology

4.1 Calibration

The parameter values used in this paper are indicated in Table 1. The time period of the model is a year. Each agent lives 39 model periods, which corresponds to a real life age of 20 to 58. The real interest rate is set to 4 percent. Remember from Proposition 1 that the rental wage of human capital is equal to \( w_j = (1 + g)^{j-1} \). \( g \) is chosen to equal the average growth rate in average real earnings per person over the period 1969-1992 in the PSID sample. As late in life little or no new human capital is produced, the depreciation rate \( \delta \) is set so that the model produces the rate of decrease of average real earnings at the end of the working life cycle. The implication is that average earnings fall late in life when growth in the rental rate of human capital is not enough to set the mean fall in human capital.

Based on the literature surveyed by Browning et al. (1999), the elasticity parameter \( \alpha \) of the human capital production function is set to three different values: 0.5, 0.7 and 0.9. The early literature on human capital technology, using restricted models, estimated values from 0.5 to almost 1.0. Nevertheless, based on the recent analysis of Heckman et al. (1998a), one could assert that the value of \( \alpha \) is more approximated to 0.9. Estimating an unrestricted model for different education and ability groups, they find a value for \( \alpha \) that varies from 0.832 to 0.945 for males attending high school and from 0.871 to 0.939 for males attending college. This finding will be important at the moment of evaluating the quantitative impact of progressive taxation on human capital accumulation.

Following Gouveia and Strauss (1994), the benchmark “effective tax function” is calibrated to reproduce the 1989 tax schedule of the U.S. and the \( \tau_2 \) parameter is set to equalize the percentage of labour income levied by the government. Based on the estimates of Mendoza et al. (1994), labour income effective tax rates are set to the 1969-1988 average of 25.9%. As these estimates incorporate all social security contributions and payroll taxes, a 10.9% is deducted from the average as the model abstracts from social security transfers. Thus, the \( \tau_2 \) parameter is set to 15%. It is important to note that, as the tax function is set constant for all the periods in the model, the model abstracts from changes taking place during the period of analysis, as the Tax Reform Act of 1986. Finally, aggregates for the economy are obtained by using the 1989 U.S. population structure.

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The authors are not able to reject the hypothesis that a neutral Ben-Porath model \((\alpha = \beta)\) describes the human capital accumulation process for persons of different ability and education groups.
4.2 Computation

The computation algorithm is explained in detail in appendix A.2. First, the optimal decision rule $h_j(h, a)$ is calculated. Then by putting a grid on learning ability and initial human capital, life-cycle profiles of human capital, hours and earnings are calculated from these grid points. Finally, the initial distribution has to be found. This initial distribution is set to be jointly, log-normally distributed. As this class of distributions is characterized by 5 parameters, the problem consists in searching over the vector of parameters that characterize these distributions so as to minimize the distance between the model and data statistics (mean, dispersion and skewness) for before-tax earnings.\(^\text{10}\)\(^\text{11}\)

Graphically, Figure 2 shows that the mean earnings fit for $\alpha = 0.7$ generates a smoother hump shape than the observed in the PSID data, while the one for $\alpha = 0.9$ generates a more pronounced one. The dispersion fit for $\alpha = 0.7$ is almost indistinguishable from the PSID data until the age of 45, while for $\alpha = 0.9$ the dispersion at the end of the life-cycle is closer to the PSID data. For both values of $\alpha$ the skewness is bigger than the observed in the PSID. As a measure of goodness of fit, Table 2 presents the average percentage deviation, in absolute terms, between the model implied statistics and the data.\(^\text{12}\) Comparing the results obtained introducing the benchmark progressive schedule with the ones obtained in the parametric approach of Huggett et al. (2006), results are quite similar. Thus, the approach followed in this paper is also able to replicate the qualitative properties of the U.S. earnings distribution dynamics.

Referring to the properties of initial distributions, the statistics are presented in Table 3 for different values of $\alpha$. First of all, correlation between ability and initial human capital is positive and important. Following Proposition 1, given a fixed level of initial human capital, high ability agents produce more human capital than low ability ones. Then, this result is implied by the dynamics of the model.\(^\text{10}\) These parameters are the mean ($\mu_a$) and the standard deviation ($\sigma_a$) for ability, the mean ($\mu_{hc}$) and the standard deviation ($\sigma_{hc}$) for initial human capital and the correlation ($\rho$) between ability and initial human capital.\(^\text{11}\) Formally, the problem consists in look for the vector of parameters $\gamma = \{\mu_a, \mu_{hc}, \sigma_a, \sigma_{hc}, \rho\}$ that minimizes the distance between the model and data statistics:

$$\min_{\gamma} \sum_{j=1}^{J} \left\{ \log \left( \frac{m_j}{m_j(\gamma)} \right)^2 + \log \left( \frac{d_j}{d_j(\gamma)} \right)^2 + \log \left( \frac{s_j}{s_j(\gamma)} \right)^2 \right\}$$

where $m_j$ denotes the mean, $d_j$ the standard deviation and $s_j$ the skewness for the PSID data and $m_j(\gamma)$, $d_j(\gamma)$ and $s_j(\gamma)$ the corresponding model statistics. This form of the objective function ensures that the numerical solution to the problem is not affected by the units of measurement of the statistics in question.\(^\text{12}\) The measure of goodness of fit is given by:

$$\left[ \sum_{j=1}^{J} \left\{ \log \left( \frac{m_j}{m_j(\gamma)} \right) + \log \left( \frac{d_j}{d_j(\gamma)} \right) + \log \left( \frac{s_j}{s_j(\gamma)} \right) \right\} \right] / (3J)$$

\(^{10}\)These parameters are the mean ($\mu_a$) and the standard deviation ($\sigma_a$) for ability, the mean ($\mu_{hc}$) and the standard deviation ($\sigma_{hc}$) for initial human capital and the correlation ($\rho$) between ability and initial human capital.

\(^{11}\)Formally, the problem consists in look for the vector of parameters $\gamma = \{\mu_a, \mu_{hc}, \sigma_a, \sigma_{hc}, \rho\}$ that minimizes the distance between the model and data statistics:

\(^{12}\)The measure of goodness of fit is given by:
Secondly, as the parameter $\alpha$ increases, the mean and the standard deviation of ability decline while the opposite is true for the mean and the standard deviation of initial human capital. As the parameter $\alpha$ represents a scaling parameter of the returns to human capital, as this value increases, it turns out to be more profitable to invest more time in human capital production early at life, resulting in an increase in end of life human capital. In addition to this, a higher value of $\alpha$ implies a more "efficient" human capital production function, thus reinforcing the aforesaid effect. Therefore, for given learning ability and initial human capital a higher value of $\alpha$ lowers earnings early in life and rises earnings later in life. It is worth noting that lowering mean ability and rising mean initial human capital serves to counteract the effect of increasing $\alpha$. However, as $\alpha$ is a technology parameter, specifically an elasticity, changing its value does not only reflect on earnings levels, but on marginal decisions and in the economy structure as was mentioned above. Then, increasing the value of $\alpha$ will have more impact than just rotating earnings profiles, the latter being the effect of changing mean ability and mean initial human capital. Finally, the increase in $\alpha$ has a fundamental effect over the ability standard deviation, causing that the initial distribution becomes much more concentrated toward the mean, thus reducing the relative importance of initial conditions for lifetime inequality.

5 Results

5.1 Toward a proportional schedule

Table 4 presents the main findings of this paper. It accounts for the change from the benchmark progressive tax schedule to a proportional schedule maintaining unchanged the percentage of labour income levied by the government. First of all, qualitative results are not sensitive to the elasticity parameter of the production function. In all cases time invested and human capital increase. As earnings are proportional to human capital, having more time invested in human capital production early in life leads to higher earnings at the end of the life cycle, shifting the curve of mean earnings. Even though this investment in human capital entails lower earnings at the beginning of life, the present value of after-tax earnings increases, implying a benefit for the economy on average. Although there is a benefit for the economy on average, the different measures of inequality increase. A priori this result was expected because of the dynamics of the model. Facing lower marginal and effective taxes under the proportional schedule, high ability agents experience an important change on their lifetime income, especially at the end of their lives. On the other hand, low ability agents -those having low earnings- have to pay more taxes under the proportional schedule and in addition to this, they do not experience an important increase in their end of life earnings. As the raise in time invested early in life, induced by the change of schedule, leads to small rises in end of life earnings, these are offset by the discounting factor when taking into account the present value of earnings. Thus, it is clear that the
existing trade-off leads to winners and losers when progressivity is decreased.

Referring to the quantitative impact, as the elasticity parameter tends to one, the effect becomes more important. For instance, when the values of \( \alpha = 0.5 \) and \( \alpha = 0.9 \) are considered, the change in time invested varies from 2.21% to 10.65%, resulting in a change in after-tax earnings from 0.31% to 8.31% and a change in the Gini coefficient from 7.14% to 25.21%. These results are in line with the consequences a higher value of \( \alpha \) has on the model, as was discussed at the end of the previous section. Thus, assessing a value for \( \alpha \) is significant when evaluating the quantitative effects of the change of schedule. As was mentioned in Section 4.1, several studies have tried to estimate the value of the elasticity parameter. In particular Heckman et al. (1998a), besides being one of the most recent in the Ben-Porath production function literature, estimate an unrestricted model with heterogeneity in human capital and skills. Their estimates for the elasticity parameter range from 0.871 to 0.939 for males attending college.\(^{13,14}\) Since the form of heterogeneity and the criteria for the selection of the sample are similar to the ones in this paper, for the remainder of the paper results will be based on the value of 0.90 for the elasticity parameter of the human capital production function.\(^{15,16}\)

Figure 3 represents the earnings dynamics under the two tax schedules. Results are striking in that a decrease in progressivity could translate in an increase of 8.31% in after-tax earnings and of 7.08% in the present value of after-tax earnings for the economy on average. These values imply an important difference with the results obtained by Heckman et al. (1998b) and Li and Sarte (2004), as in the former the change on aggregate output is -0.08% under partial equilibrium and 1.15% under general equilibrium, while in the latter the change is between 0.12% and 0.34%. Nonetheless, Li and Sarte (2004) estimate the change in the Gini coefficient ranges between 20% and 24% when progressivity is reduced, while in this paper the change is 25.21%. The significant increase in all measures of inequality caused by a less progressive schedule is opposed to the findings of Caucutt et al. (2003) and Caucutt et al. (2006).

In Figure 3(b), there is a much more pronounced U-shaped pattern for earnings under the proportional schedule than under the progressive one. Under both schedules the mechanism is the same, though its results are reinforced under the proportional schedule. As early in life high ability agents devote most of their time or all available time to accumulating human capital, their earnings are lower than those of their low ability counterparts. Under the progressive schedule, the bottom of the U-shape occurs at about age 25 (27 for the proportional one)
when earnings of high ability agents overtake those of lower ability agents. After this age, earnings dispersion increases as high ability agents have more steeply sloped age-earnings profiles than low ability agents. This mechanism also produces an increase in the skewness as depicted in Figure 3(c). Since it is clear that progressivity helps reducing inequality at all ages, in the next section we will focus on how progressivity affects different types of agents.

5.2 Time invested and earnings

Figure 4 outlines the optimal decision rule for time invested in human capital accumulation and before-tax earnings over the life-cycle for two different agents. The first one (panels (a) and (c)), is a low ability agent which has a low level of initial human capital, henceforth denoted as the “unfortunate” agent. The other one (panels (b) and (d)) is a “fortunate” agent, as he has high learning ability besides having a high level of initial human capital. Hence the analysis will focus on the extremes of the agents’ distribution.

Consequences of a progressive schedule seem clear. In both cases, agents invest more time in education under the proportional schedule. However, investment profiles are quite different. The “unfortunate” agent never invests in full-time education and the fraction of time allocated in human capital accumulation rises from 20% to 22%. His after-tax earnings under the progressive schedule are always higher than under the proportional schedule since taxes paid change from a 8.70% to a 15%. In addition to paying higher taxes under the proportional schedule, the increase in time invested early in life implies a decrease in earnings early at life. Besides, the rise in earnings at the end of the life-cycle due to investing in more human capital accumulation is offset by the discounting factor when taking into account the present value of earnings. Consequently, under the proportional schedule, the effects of investing more time in human capital accumulation and paying more taxes cause a reduction of 7.01% in the present value of his after-tax earnings relative to the progressive schedule.

On the other hand, under the proportional schedule the “fortunate” agent spends the first ten years of his life in full-time education, while under the progressive schedule only the first three. Even though the evolution of time invested is smoother under the proportional schedule, the consequences of studying full-time seven years more are striking. The slope of the earnings profile is much steeper, clearly suggesting the effects of ability and human capital in earnings at the end of the life-cycle. The increase in the present value of after-tax earnings caused by the change of schedule is the shocking figure of 84.63%, reflecting the aforementioned effect of investing more time and because taxes paid shrink from 24.44% to 15% taking into account the last twenty years of his working life. It is apparent that the change of schedule has a negative impact on the “unfortunate” agent and a very positive on the “fortunate” agent. However, these agents were “extremes”. The next section accounts for those who benefit from a decrease in progressivity and those who do not.
5.3 Winners and losers

Figure 5 represents the change in the present value of after-tax earnings for all agents in the economy. First of all, it should be noticed the significant correlation between ability and initial human capital that was mentioned in Section 4.2. Agents, whose present value of after-tax earnings is lower under the proportional schedule than under the progressive one, are depicted in red while those who are better off are depicted in blue.

The area depicted in blue is larger than the one in red, what was expected due to the results obtained in Table 4, as the average agent faces an increase of 7.08% in the present value of after-tax earnings. As the previous section elucidated, agents having an above-average learning ability and initial human capital see their present value of earnings rise. Nevertheless, the increase is not identical; those who are mostly benefited are agents located in the “north-east” of the figure. On the other hand, agents located in the “south-west” of the figure are those who will be more reluctant to a change of tax schedule. In particular, the median agent faces a decrease of 2.01% in his present value of after-tax earnings. Therefore, any attempt to change the tax schedule should take this into account and compensate (some of the) agents who are worse off in order to successfully switch the tax schedule.

5.4 Earnings inequality

In Section 3.2 some key facts of the dispersion of the earnings distribution were described, specially the fanning out of the distribution. Figure 6 plots the age percentiles of earnings under both schedules. The pattern of both panels should now be familiar to the reader. In both cases the Gini coefficient increases with age (Figure 3(b)) as a result of the steeper sloped age-earnings profiles of high ability agents explained before, and as expected, the effect is more important under the proportional schedule. The strong fanning out of the highest percentile, is the result of the dynamics of the model for agents similar to the “fortunate” agent of Section 5.2, implying a shift in the skewness for all ages, specially at the end of the life-cycle, under the proportional schedule relative to the progressive one (Figure 3(c)). Hence, a decrease in progressivity raises human capital investment and earnings for the economy on average at the cost of higher inequality. In particular, the Gini coefficient for after-tax earnings shifts from 0.3023 to 0.3785.17

5.5 Horizontal versus vertical equity

So far the analysis was mainly focused on the comparison between after-tax earnings for different agents under both tax schedules. Specifically, the principle of

17The Gini coefficient for before-tax earnings under the progressive schedule is 0.321 which is similar to the value of 0.330 found in Huggett et al. (2006) and of 0.351 in Castaneda et al. (1998)

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vertical equity was clearly reflected under the progressive schedule as agents with greater ability to pay taxes -agents with high ability and/or high initial human capital- did so. All the same, it seems important to analyze if there is an arbitrary discrimination in regards to horizontal equity. For that reason, two agents with similar present value of before-tax earnings are considered. As both agents have similar present value of earnings, based on the horizontal equity principle one would expect that the progressive schedule should treat both in an equal manner.

Still, these agents differ in their innate ability and initial human capital. The first one, is a relative low ability agent having a high level of initial human capital. The other one, is a relative high ability agent having a low level of initial human capital. Figure 7 depicts the evolution of before-tax and after-tax earnings over the life-cycle for both agents. Based on the dynamics of the model, results are not surprising. The progressive schedule negatively affects both agents -one has high ability and the other one high human capital-. However, horizontal inequity is surprisingly apparent when annual income is taxed, as after-tax earnings from a progressive schedule to a proportional one increase 2.17% for the low ability agent and 49.59% for the high ability one. This result would imply that it is important to consider lifetime income as an alternative measure to compare taxpayers rather than using annual income, as the horizontal equity principle is not always applied when the vertical equity is.

6 Conclusions

Under an alternative framework than the one generally used in the literature mentioned in Section 1, this paper attempts to analyze the effects of progressive income taxes over human capital accumulation. As the model is able to replicate the qualitative properties of the U.S. earnings distribution dynamics, it seems quite reasonable to use this model as a benchmark. As agents with steeper sloped earnings profiles are those who start off with high learning ability and/or initial human capital, one would expect that these agents are the most affected by a progressive schedule. Nevertheless, the most affected agents are those born with high learning ability, even if they have a low level of initial human capital, thus making asymmetric the impact over human capital accumulation and, hence, earnings.

In particular, if the progressive schedule is changed to a proportional one, time invested in the economy rises as the distortionary effects of taxation over the decision rules are reduced. In addition to increasing human capital production, mean after-tax earning rises, as well as the present value of after-tax earnings. However, there is no “free lunch” as the “efficiency-equality” trade-off becomes apparent. All measures of inequality rise in an important manner, basically because of the fanning out of the highest percentiles of the earnings distribution. As earnings at the end of the life-cycle are proportional to human capital, agents with high

\footnote{18One could associate the former as a below average ability agent grown in a “prosperous” family and the latter with a “brilliant” agent grown in a not so favorable environment.}
ability and high initial human capital are those who are mostly benefited by the change of tax schedule. Even agents with a low level of initial human capital would prefer the proportional schedule conditioned that they also are high ability agents. Then, the success of an hypothetical reform toward a proportional schedule depends on the proportion of winners and losers in the economy, which depends on the conditions of the initial distribution.

It is important to note that not only the level of taxes affects the results for economy but the structure of the tax schedule. Specifically, in regards to equity, the progressive schedule responds to the vertical equity principle. While different standards of tax fairness may give different answers about how people with different incomes should be taxed, tax burdens should be distributed in a manner that is horizontally equitable. Thus, using lifetime income as a measure for comparing taxpayers could lead to horizontal equity as well as to a reduction in the inefficiencies created by annual income progressive taxation.

Finally, there is one important subject that was left without consideration. The analysis made was mostly quantitative, as it focused on the variation in time invested and in human capital accumulation caused by a change of the tax schedule. Nevertheless, the qualitative dimension is also important. One of the main results was that in a less unequal economy, agents invest less time in full-time education and this generated a reduction in human capital accumulation. However, the opposite is true. One could argue that an economy having most of its population studying full-time until their 24 years, would mean that the educational level achieved by its inhabitants is quite high. Then, this investment in human capital would have a positive impact on the economy long-run growth rate, increasing the welfare of its inhabitants. It seems the choice lies in having less inequality today versus larger welfare tomorrow.
References


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A Appendix

A.1 Proposition 1

Proof.

(a) The continuity of the value function follows by repeated application of the Theorem of the Maximum starting in the last period of life. To show that the value function increases in \( h \) and \( a \), this must hold in the last period. Since \( V_J(h, a) = (1 - \tau_0)w_Jh + \tau_0 \left[ (w_Jh)^{-\tau_1} + \tau_2 \right]^{-\frac{1}{\tau_1}} \), restrictions over the tax function parameters must be set. Sufficient conditions for this are: \( 0 < \tau_0 < 1 \) and \( \tau_2 > 0 \). For the sake of completeness, let \( \tau_1 > 0 \) or \( \tau_1 = -1 \). Backward induction establishes the result for earlier periods using the fact that after-tax earnings increase in \( h \) and \( a \).

Under these conditions, the concavity of the value function in human capital follows from backward induction.\(^{19}\) First, the human capital production function is concave in current human capital. Second, after-tax earnings are jointly concave in \((h, h')\).\(^{20}\) Third, the terminal value function \( V_{J+1}(h, a) \equiv 0 \) is concave in human capital.

The decision rule \( h_j(h, a) \) is single-valued since the objective function is strictly concave and the constraint set, for given \((h, a)\), is convex. The objective function is strictly concave because the value function is concave and because after-tax earnings are strictly concave in \( h' \).

(b) The result yields by inserting the human capital production function into the law of motion for human capital accumulation and solving for \( l \).

(c) Derived by solving the Bellman equation. There is no closed-form solution though the same conclusions as in Huggett et al. (2006) can be drawn for the sufficient conditions set for the tax function.

\(^{19}\)See for instance Stokey et al. (1989) and Ljungqvist and Sargent (2004).
\(^{20}\)It can be shown that the statement holds for the tax function sufficient conditions set before.
A.2 Computational algorithm

To calculate the optimal decision rule \( h_j(h, a) \), a uniform grid of 5,300 points on human capital is put on \([0, h^\ast]\) as well as a uniform grid of 20 points on ability on \([0, a^\ast]\), where \( h^\ast \) and \( a^\ast \) are determined by the guess of the initial distribution. This initial distribution is set to be jointly log-normally distributed. For these grid points, the optimal decision rule is calculated by backward induction starting at period \( J - 1 \) as

\[
V_J(h, a) = (1 - \tau_0)w_Jh + \tau_0 \left[ (w_Jh)^{-\tau_1} + \tau_2 \right]^{\frac{1}{1 - \tau_1}} \quad \text{and} \quad V_{J+1} = 0.
\]

Following Judd (1999), in order to gain in efficiency, rather than solving by value function iteration, the bisection method is used to solve the Euler Equation (6) for every grid point for \( j = 1, \ldots, J - 1 \). Having solved the decision rule, the life-cycle profiles of human capital, hours and before-tax earnings are simulated for these grid points.

Afterward, a random sample of 10,000 ability and human capital bins determined by the initial distribution is drawn. As these values are not restricted to lie on these grid points, use linear interpolation is used to calculate values off grid points. Using the Nelder-Mead method, the vector of parameters describing the joint log-normal distribution that minimizes the distance between model and data statistics is looked for.\(^{21}\) For any trial of the vector, the mean, dispersion and skewness statistics are calculated at each age using the calculated life-cycle profiles and the guessed initial distribution.

\[^{21}\text{The method is described in Press et al. (1992).}\]
### Tables

#### Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Model Periods</th>
<th>Interest Rate</th>
<th>Rental Growth Rate</th>
<th>Depreciation Rate</th>
<th>Production Function</th>
<th>Tax Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J=39$</td>
<td>$r=0.04$</td>
<td>$g=0.0014$</td>
<td>$\delta=0.0114$</td>
<td>$\alpha = {0.5, 0.7, 0.9}$</td>
<td>$\tau_0 = 0.258, \tau_1 = 0.768$</td>
</tr>
</tbody>
</table>

#### Table 2: Mean Absolute Deviation (%)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 0.7$</th>
<th>$\alpha = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Progressive taxes</td>
<td>7.1</td>
<td>5.0</td>
<td>6.9</td>
</tr>
<tr>
<td>Huggett et al. (2006)</td>
<td>6.8</td>
<td>5.2</td>
<td>6.4</td>
</tr>
</tbody>
</table>

#### Table 3: Ability and Human Capital at Birth

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 0.7$</th>
<th>$\alpha = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ($a$)</td>
<td>0.476</td>
<td>0.470</td>
<td>0.209</td>
</tr>
<tr>
<td>Standard Deviation ($a$)</td>
<td>0.304</td>
<td>0.295</td>
<td>0.075</td>
</tr>
<tr>
<td>Coef. of Variation ($a$)</td>
<td>0.639</td>
<td>0.627</td>
<td>0.357</td>
</tr>
<tr>
<td>Skewness ($a$)</td>
<td>1.176</td>
<td>1.170</td>
<td>1.058</td>
</tr>
<tr>
<td>Mean ($h_1$)</td>
<td>87.61</td>
<td>89.61</td>
<td>92.31</td>
</tr>
<tr>
<td>Standard Deviation ($h_1$)</td>
<td>40.30</td>
<td>42.30</td>
<td>44.56</td>
</tr>
<tr>
<td>Coef. of Variation ($h_1$)</td>
<td>0.460</td>
<td>0.472</td>
<td>0.483</td>
</tr>
<tr>
<td>Skewness ($h_1$)</td>
<td>1.101</td>
<td>1.106</td>
<td>1.107</td>
</tr>
<tr>
<td>Correlation ($a, h_1$)</td>
<td>0.620</td>
<td>0.622</td>
<td>0.781</td>
</tr>
</tbody>
</table>

**No Taxes** represents the adjustment obtained when taxes are not included in the model. **Taxes** represents the adjustment obtained when incorporating the benchmark progressive tax schedule.

#### Table 4: Towards a proportional schedule

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 0.7$</th>
<th>$\alpha = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time invested</td>
<td>+2.21</td>
<td>+3.61</td>
<td>+10.65</td>
</tr>
<tr>
<td>Human capital</td>
<td>+0.65</td>
<td>+1.40</td>
<td>+11.27</td>
</tr>
<tr>
<td>After-tax earnings</td>
<td>+0.31</td>
<td>+0.66</td>
<td>+8.31</td>
</tr>
<tr>
<td>StdDev log(After-tax earnings)</td>
<td>+3.59</td>
<td>+9.58</td>
<td>+48.63</td>
</tr>
<tr>
<td>Gini coefficient</td>
<td>+7.14</td>
<td>+8.45</td>
<td>+25.21</td>
</tr>
<tr>
<td>After-tax earnings Present value</td>
<td>+0.21</td>
<td>+0.42</td>
<td>+7.08</td>
</tr>
<tr>
<td>StdDev log(Present value)</td>
<td>+0.27</td>
<td>+0.55</td>
<td>+10.11</td>
</tr>
</tbody>
</table>

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Figures

Figure 1: Before-tax Earnings Distribution Dynamics - PSID Data

(a) Mean Earnings (Earnings at age 58 = 100)  

(b) Dispersion (Gini coefficient)

(c) Skewness (Mean/Median)
Figure 2: Before-tax Earnings Distribution Dynamics - Model

(a) Mean Earnings (Earnings at age 58 = 100)

(b) Dispersion (Gini coefficient)

(c) Skewness (Mean/Median)
Figure 3: Progressive versus Proportional schedule (α = 0.9)

These figures represent the before-tax earnings distribution dynamics for the progressive and the proportional tax schedules.

(a) Mean Earnings (Earnings at age 58 = 100)  
(b) Dispersion (Gini coefficient)  
(c) Skewness (Mean/Median)
These figures outline the optimal fraction of time spent in human capital production and after-tax earnings over the life-cycle for an agent born with low learning ability and a low level of initial human capital (low, low) and for an agent born with high learning ability and a high level of initial human capital (high, high) under the progressive and the proportional tax schedules. It is worth remembering that earnings are rescaled so that mean before-tax earnings for the economy at age 58 are equal to 100.
Figure 5: Winners and Losers ($\alpha = 0.9$)

This figure maps the change in the present value of after-tax earnings for all agents in the economy caused by switching from the benchmark progressive tax schedule to the proportional one.
This figure plots the following age percentiles of earnings: (0.05,0.10,0.15,0.25,0.50,0.75,0.90,0.95,0.99). The line corresponding to the $p^{th}$ percentile shows the level of earnings such that $p$-percent of individuals earn below this level at each age. Earnings levels are normalized so that mean before-tax earnings at age 58 are 100.

(a) Progressive schedule

(b) Proportional schedule
Figure 7: Horizontal versus vertical equity ($\alpha = 0.9$)

These figures draw the before-tax and after-tax earnings for two agents having the same present value of before-tax earnings. One agent is born with low learning ability and a high level of initial human capital (LH) and the other one is born with high learning ability and a low level of initial human capital (HL) under the progressive and the proportional tax schedules.

(a) Before-tax Earnings

(b) After-tax Earnings